# Communication for maths

## Term 2, Week 3: On the transformations of functions

# Transformations of a function $f(x)$

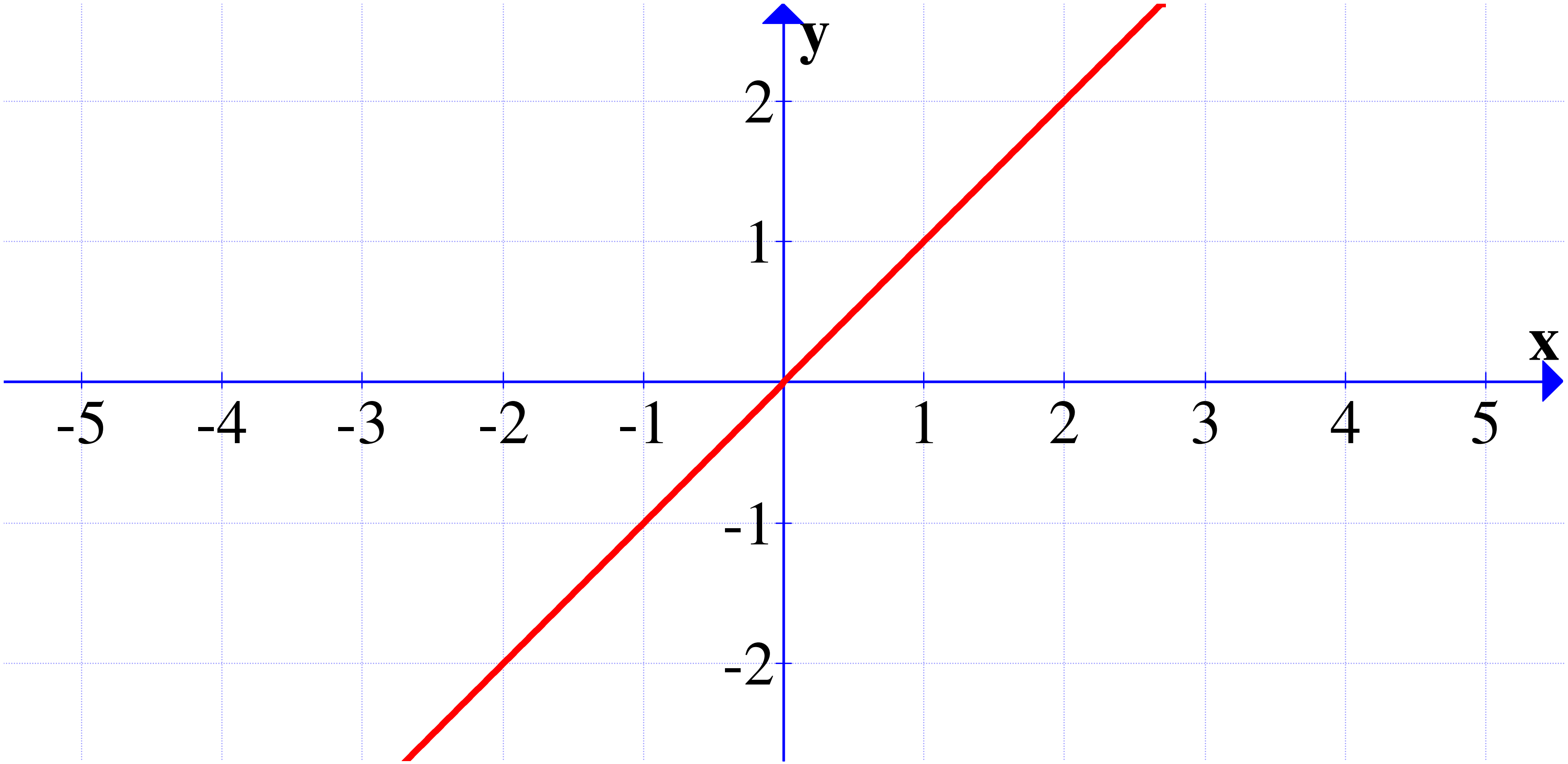
A good way to understand the effect of transformations on a function is to go through a lot of examples using graphing software, since software makes it easy and quick to change the parameters of the function you are plotting.

Use whatever software you are familiar with. The one I will use is called “Graph” and can be found for free at

http://www.padowan.dk/

# Ways of speaking

Consider the following graph of $y = mx + c$



# Ways of speaking

- An arithmetic description of this line is

  "y equals m times x plus c"

- A geometric description of this line is

  "This is a straight line of gradient m, y-intercept c and x-intercept −c/m."

# Ways of speaking

So

- *Arithmetic description* : a verbalisation of the symbols.
- *Geometric description* : a description of the mathematical meaning or effect of the transformation.

# Ways of speaking

## Examples

1) $f(x) \rightarrow a.f(x)$

"$f(x)$ gets transformed by doing $a$ times $f(x)$." **No**

"Multiply $f(x)$ values by $a$" **No**

"- - - has the effect of stretching - - -" **Yes**

# Ways of speaking

## Examples

1) $f(x) \rightarrow a.f(x)$

"$f(x)$ gets transformed by doing $a$ times $f(x)$." **No**

"Multiply $f(x)$ values by $a$" **No**

"- - - has the effect of stretching - - - in the $y$-direction"

**Yes**

# Ways of speaking

## Examples

2) $f(x) \to f(x) + a$

"Here we add $a$ to $f(x)$." **No**

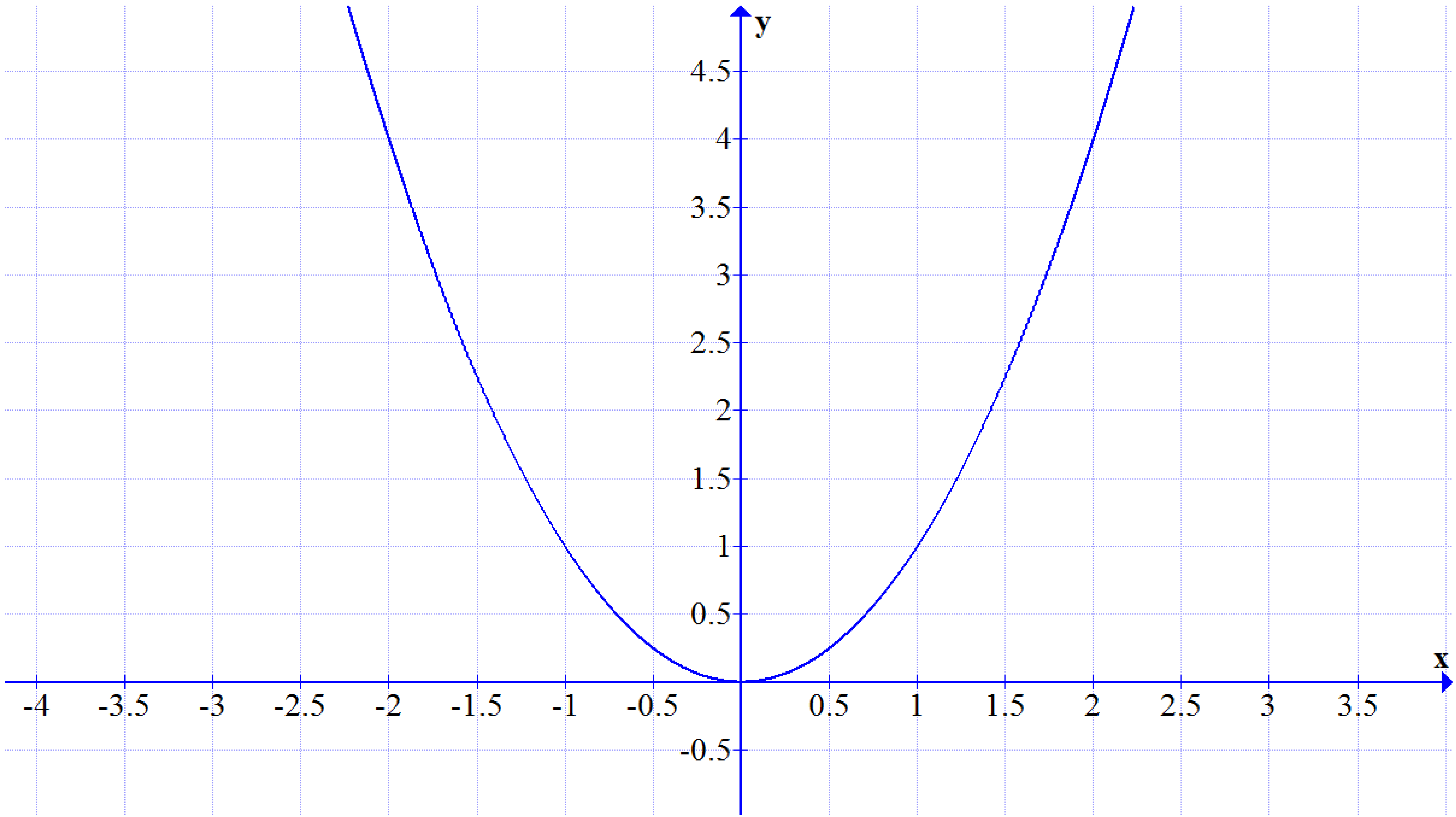
"This is $a$ plus $f(x)$" **No**

"- - - has the effect of translating - - - upwards" **Yes**

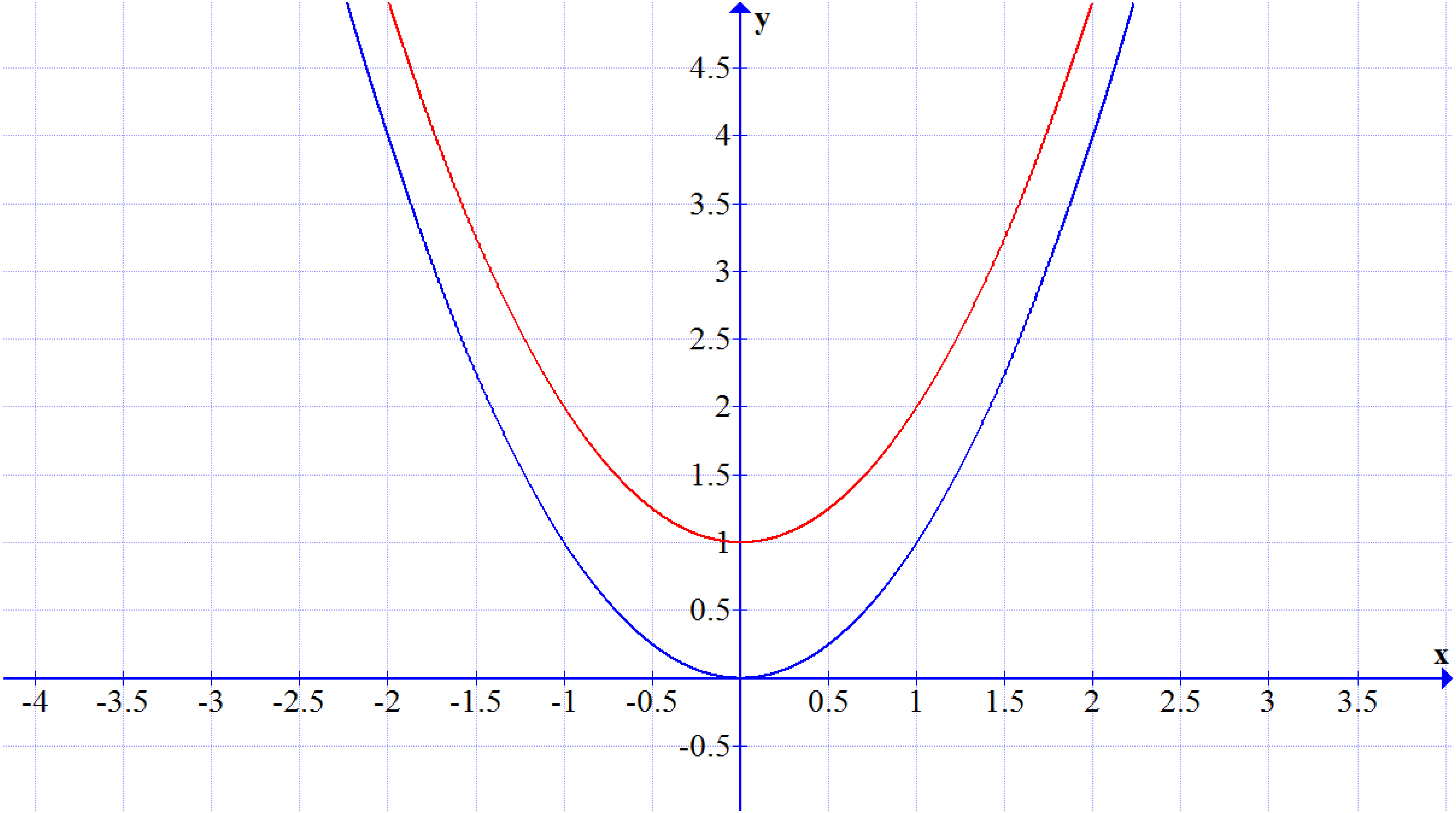
See your Ramesh/Rena's handout for more.

# Transformations: Graph 1 sequence

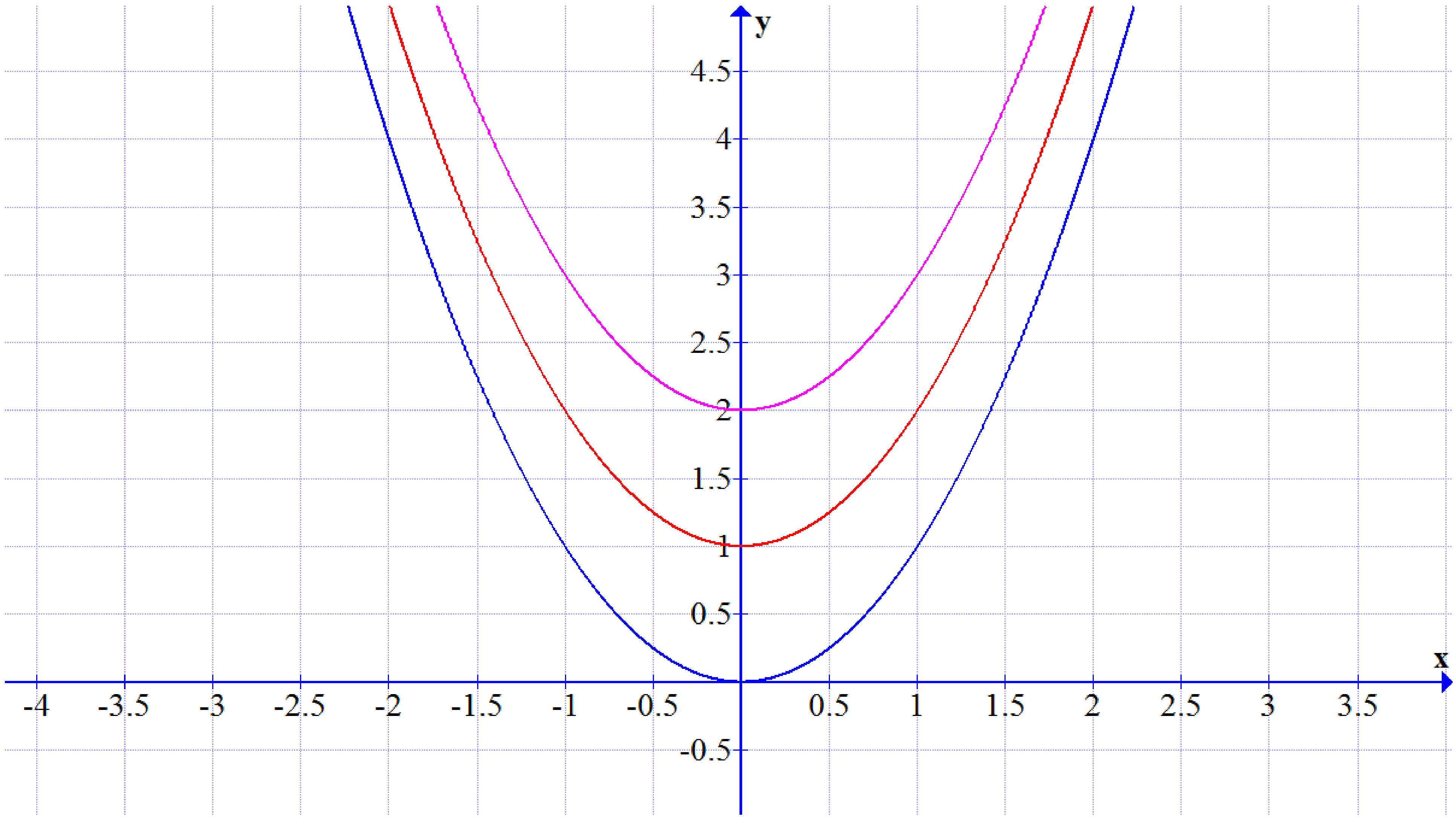
Let $f(x) = x^2$, a graph of which is shown here:



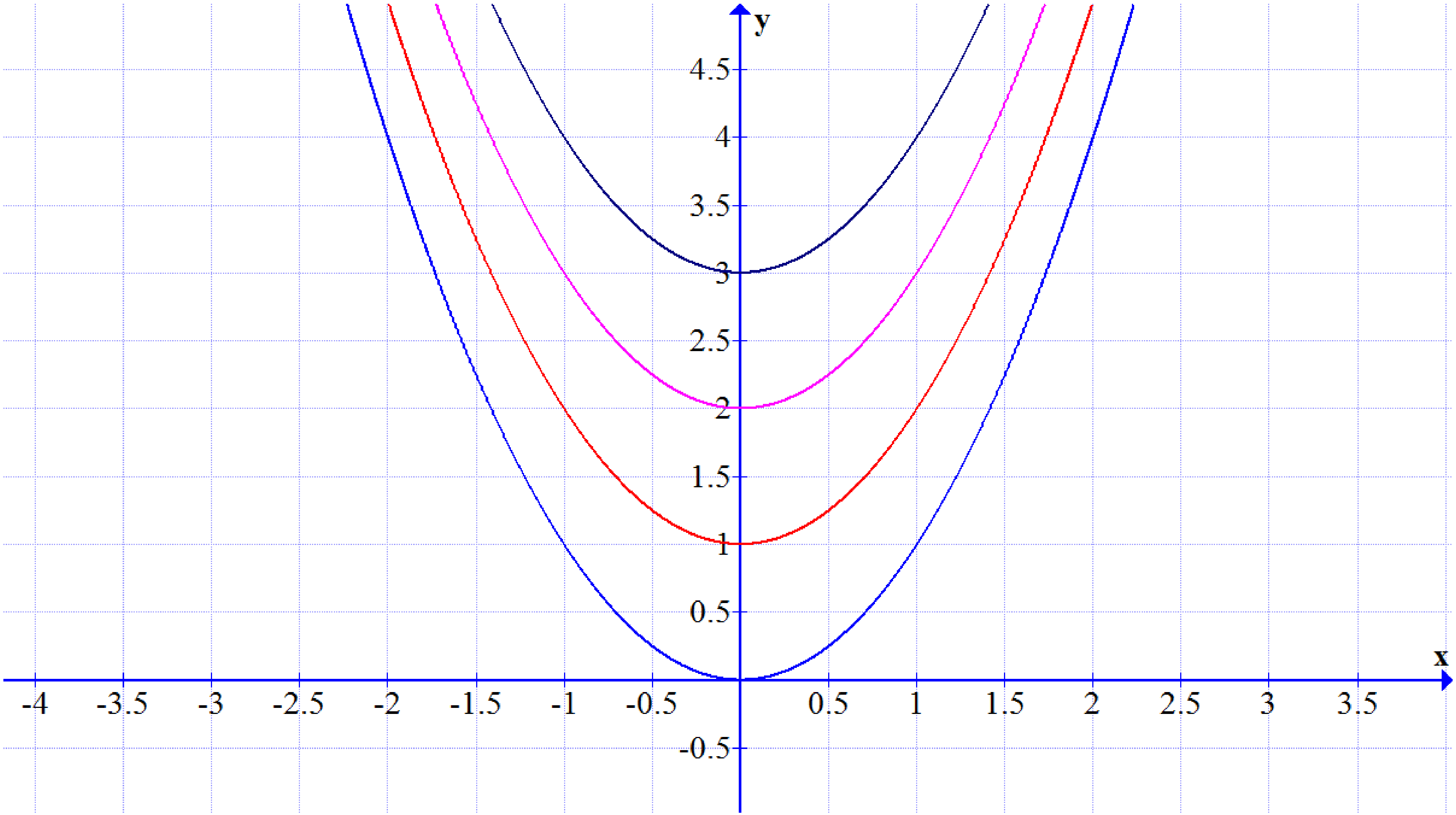
# Transformations: Graph 1 sequence



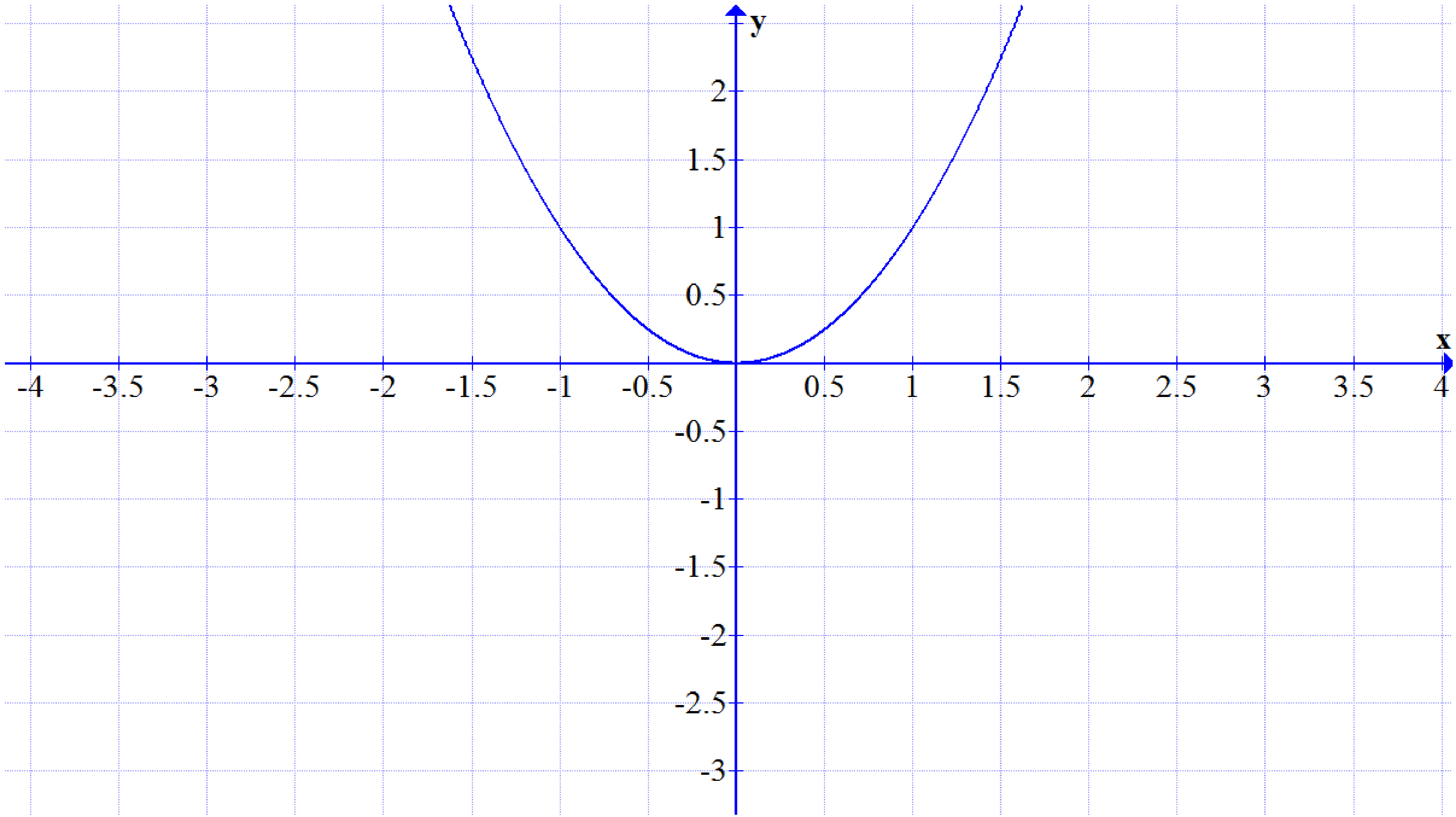
# Transformations: Graph 1 sequence



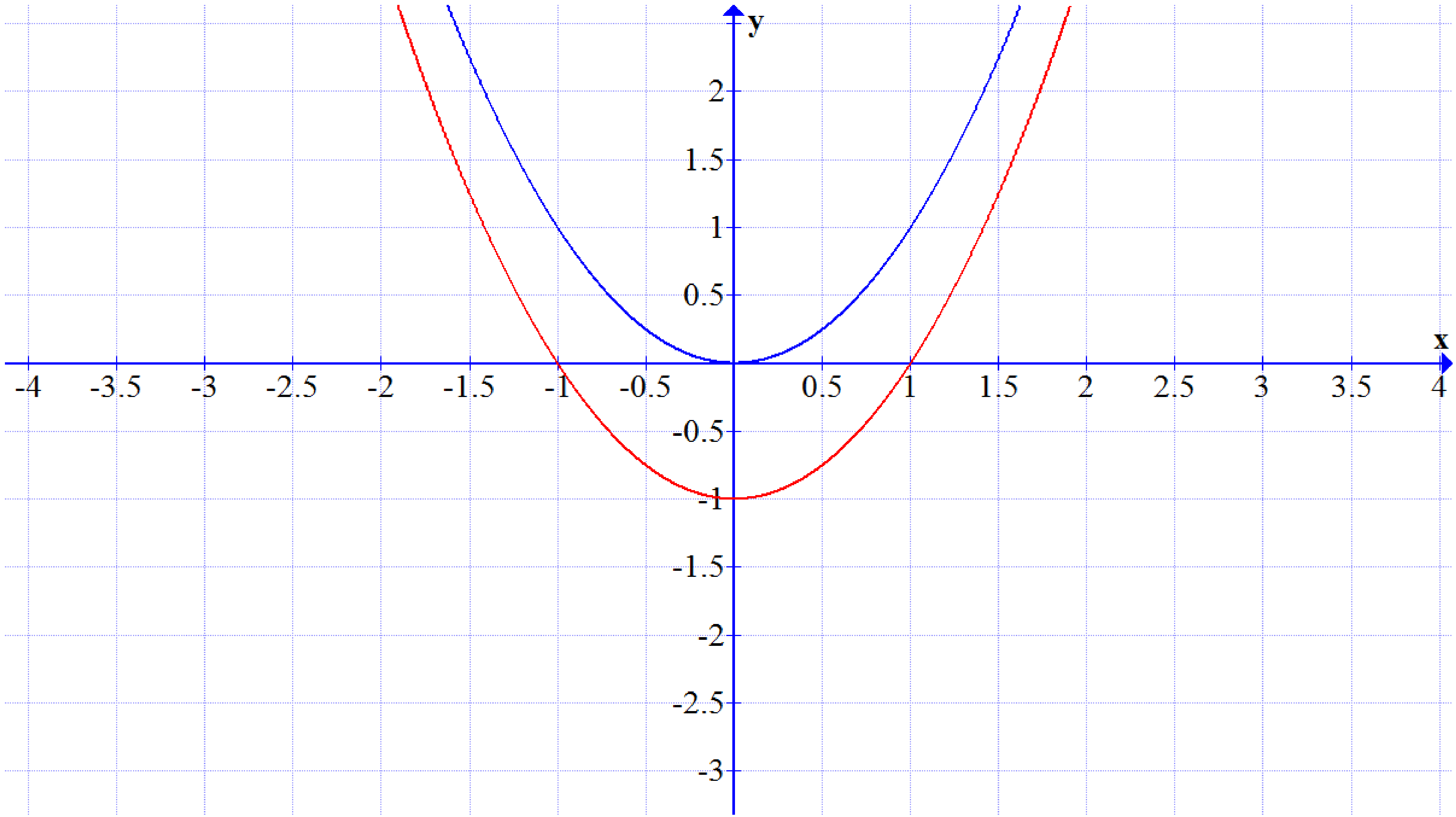
# Transformations: Graph 1 sequence



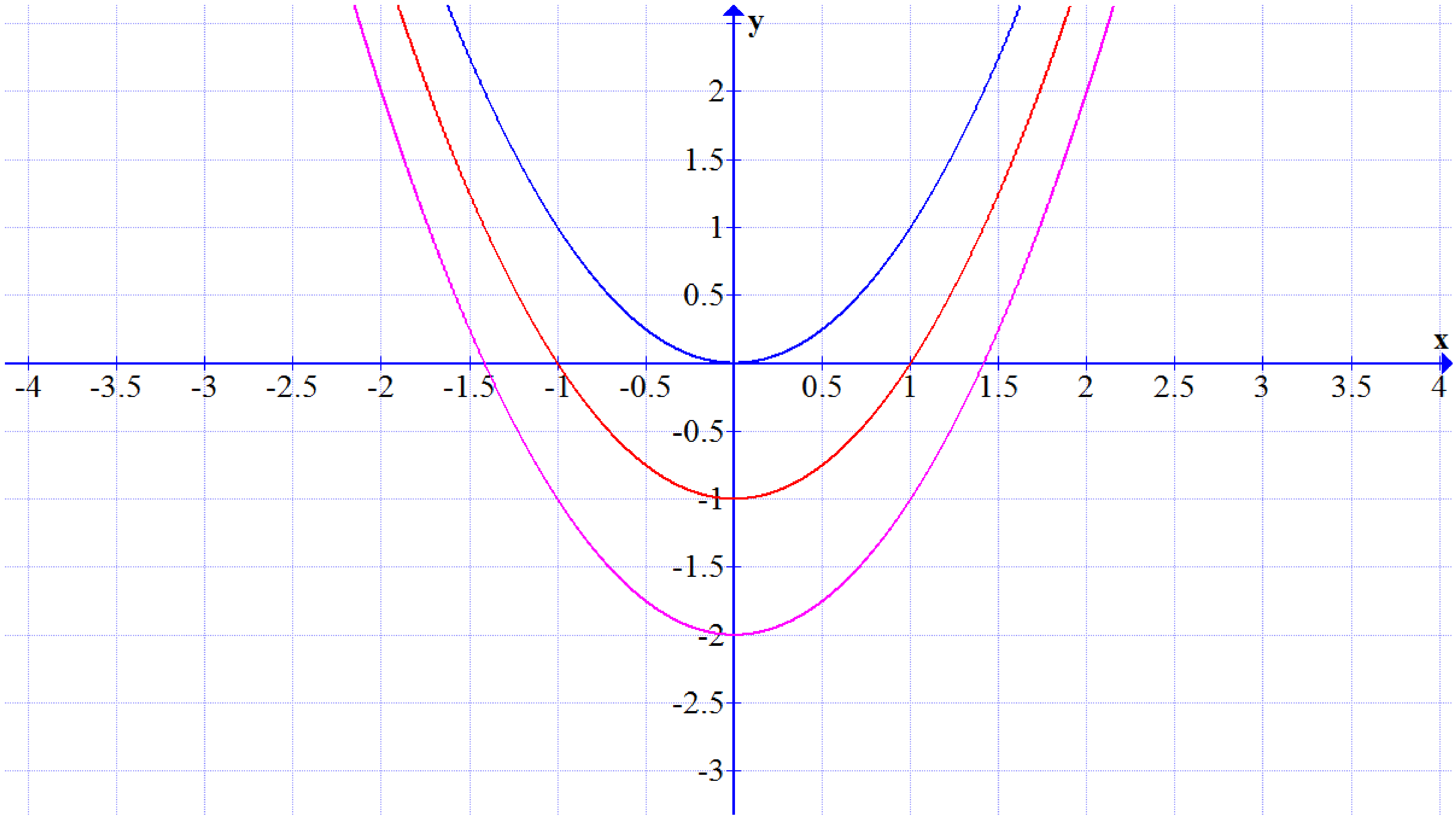
# Transformations: Graph 2 sequence



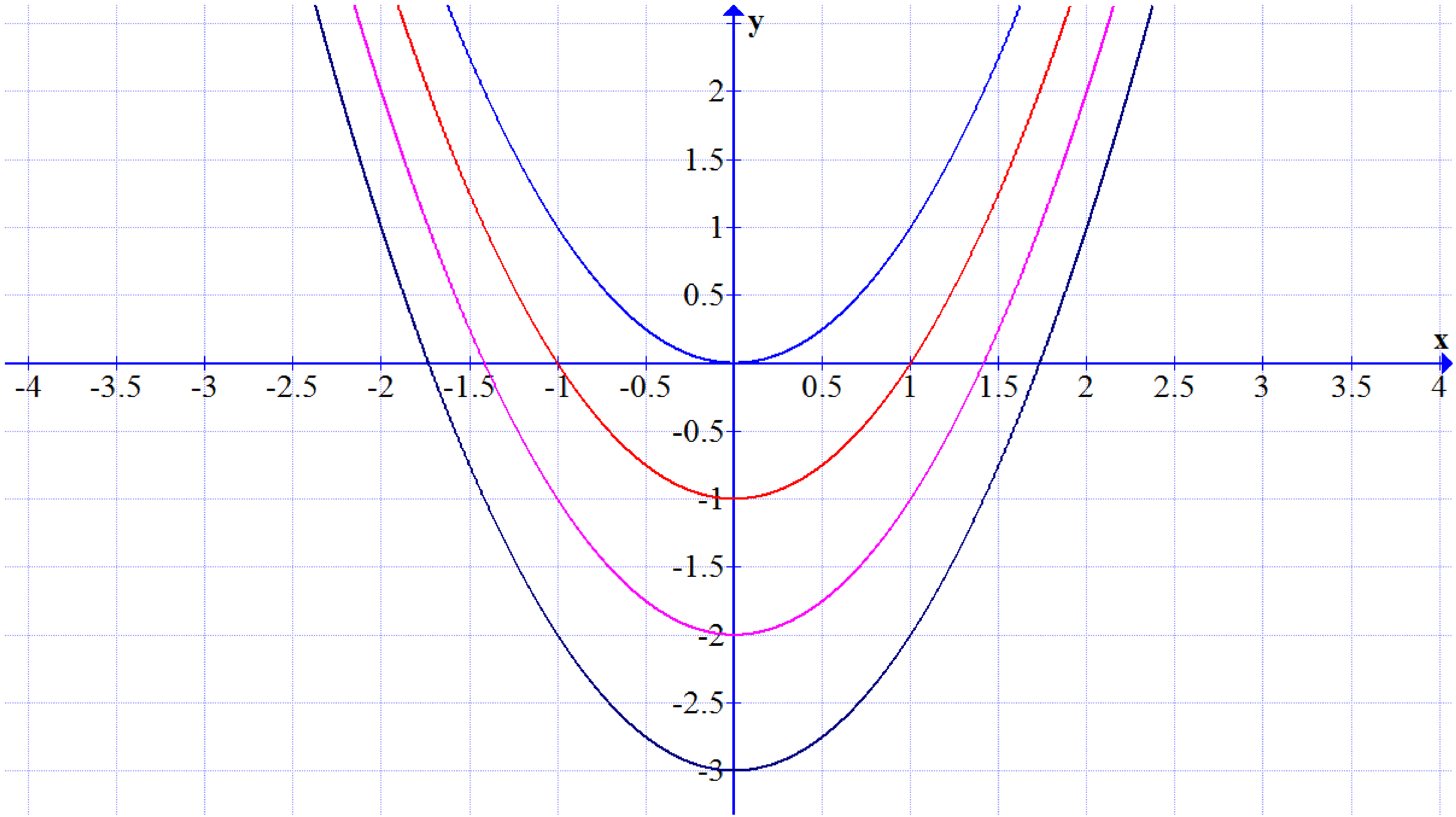
# Transformations: Graph 2 sequence



# Transformations: Graph 2 sequence



# Transformations: Graph 2 sequence



# Ways of speaking

- *Question:* How are we going to describe the effects of the transformations shown in the previous examples?
- *Answer*

  See lesson.

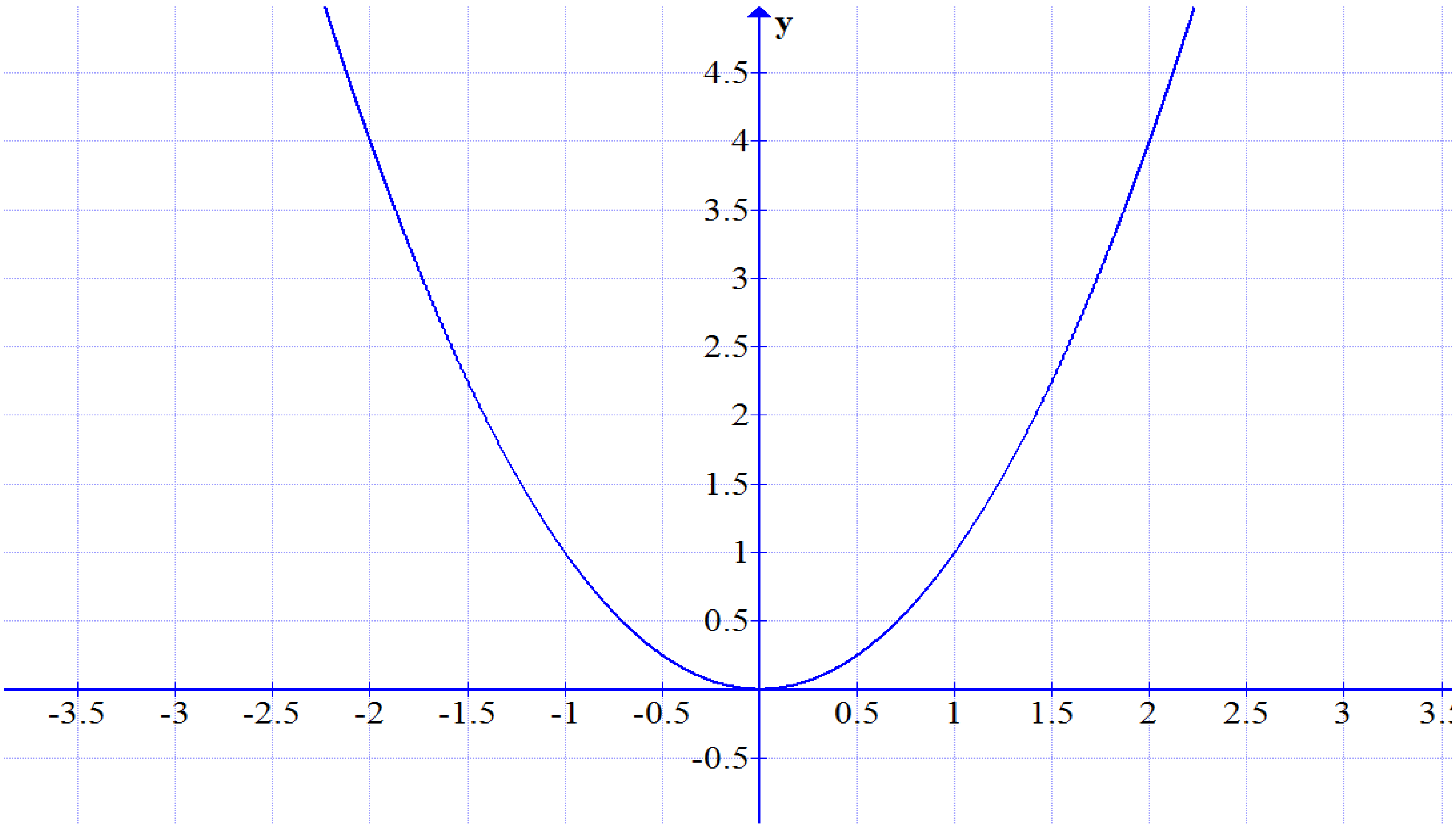
# Ways of speaking

- Some useful terminology is shown below:

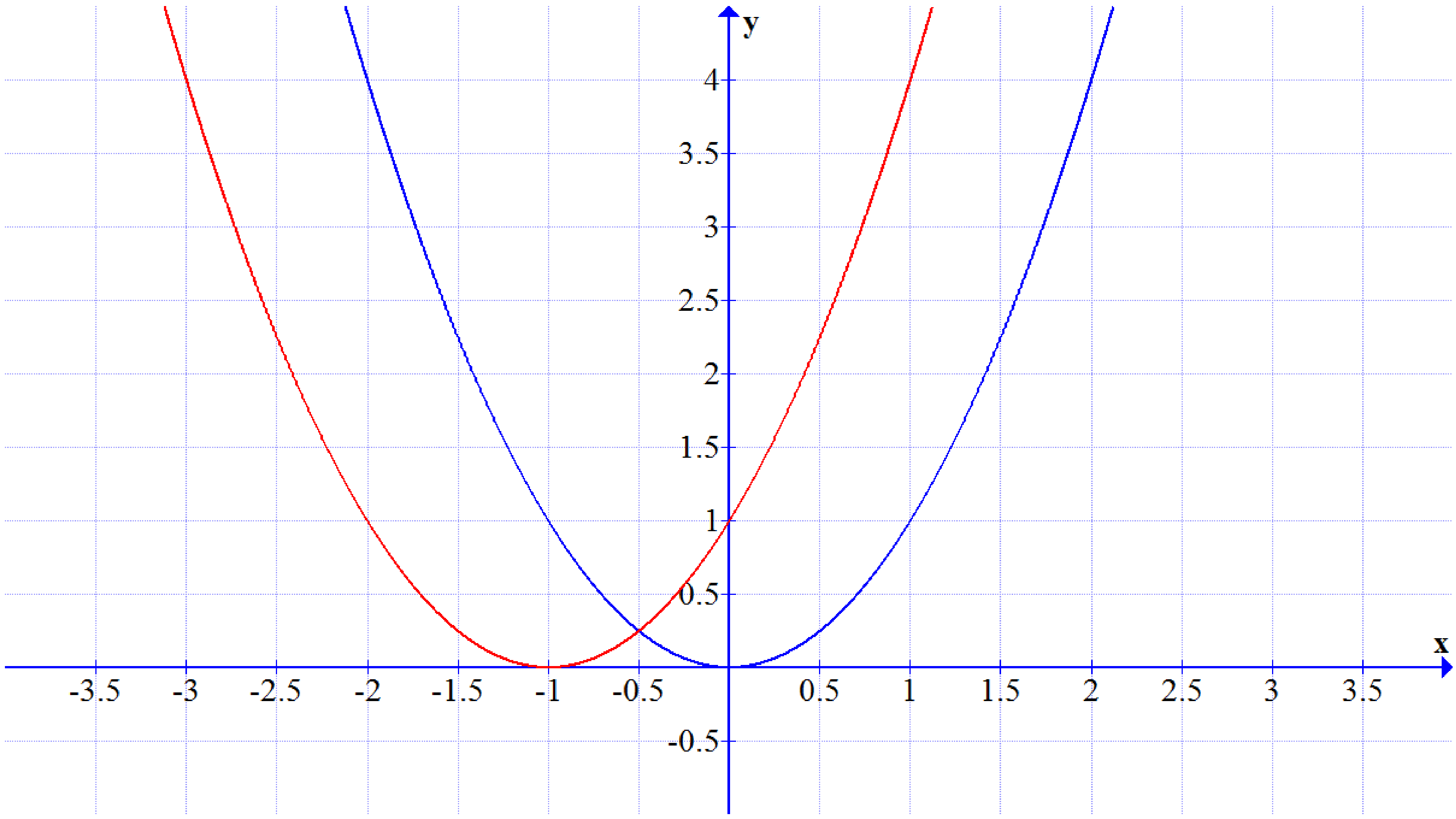
| | | |
|---|---|---|
| Stretch | Interval | Reflect |
| Differentiable | Translate<br>up/down or left/right | Continuous |
| Curve | $[a, b]$ | Squash |
| Function | | |

# Transformations: Graph 3 sequence

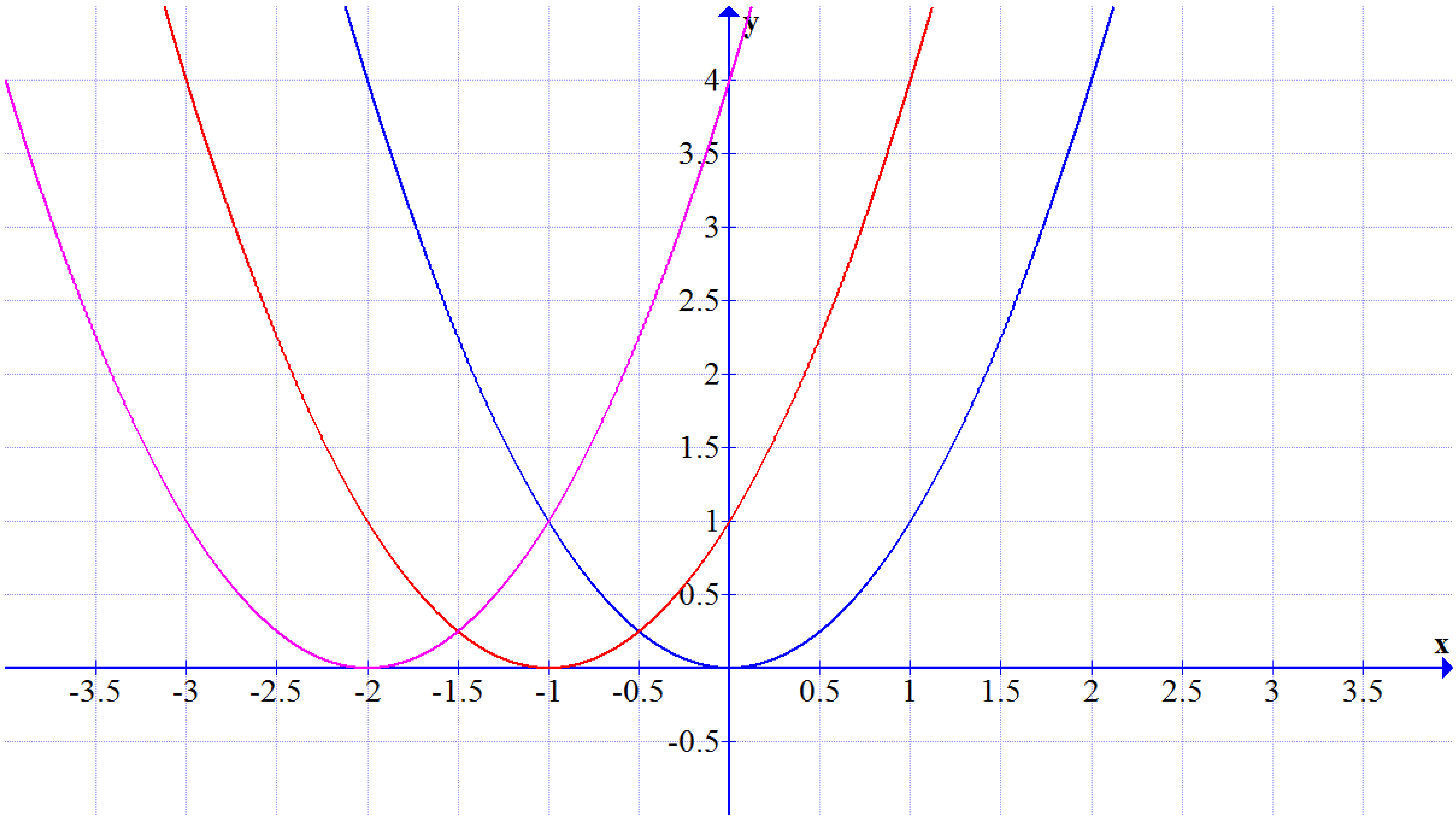
Let's look again at the function $f(x) = x^2$, a graph of which is shown here



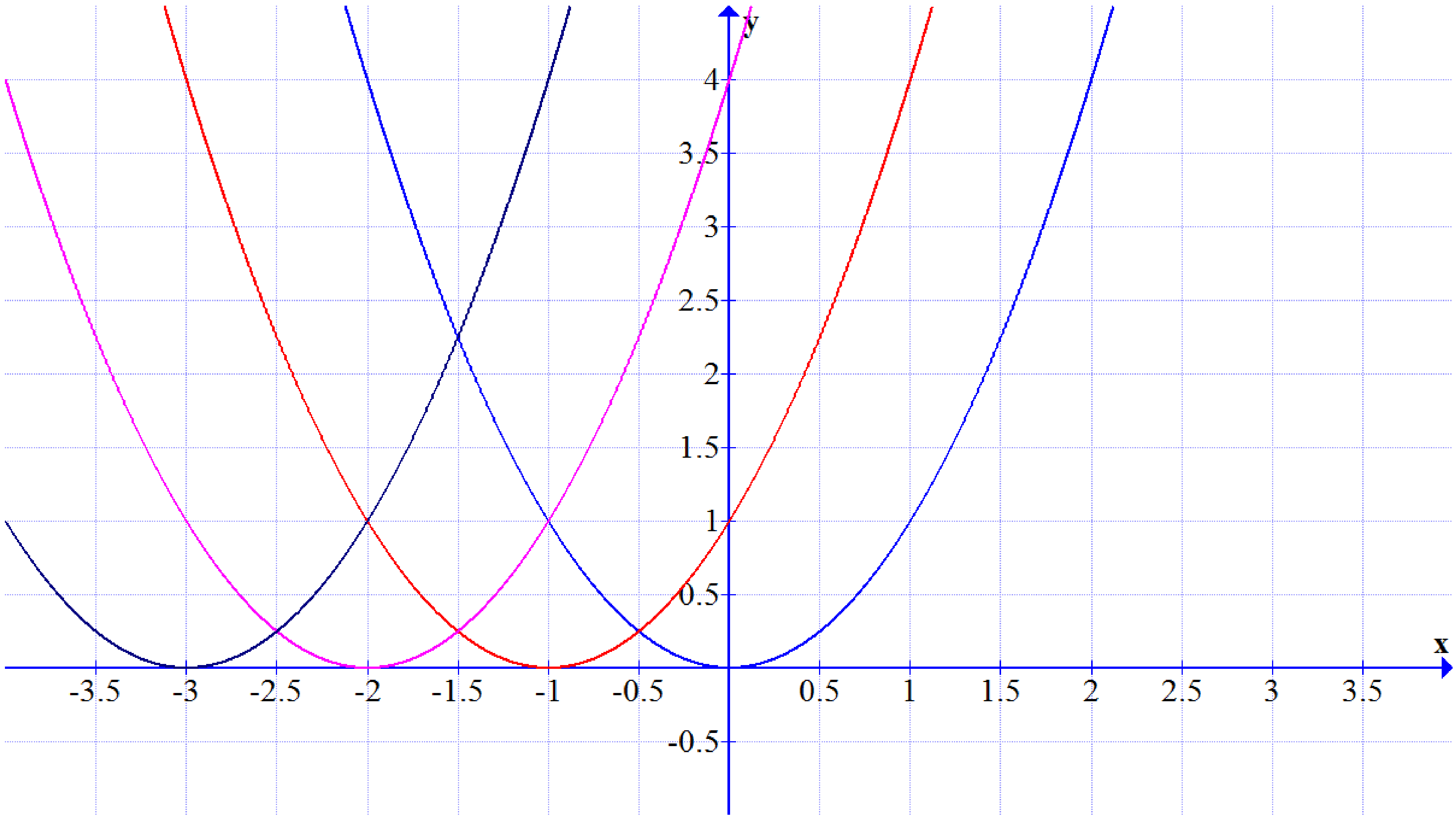
# Transformations: Graph 3 sequence



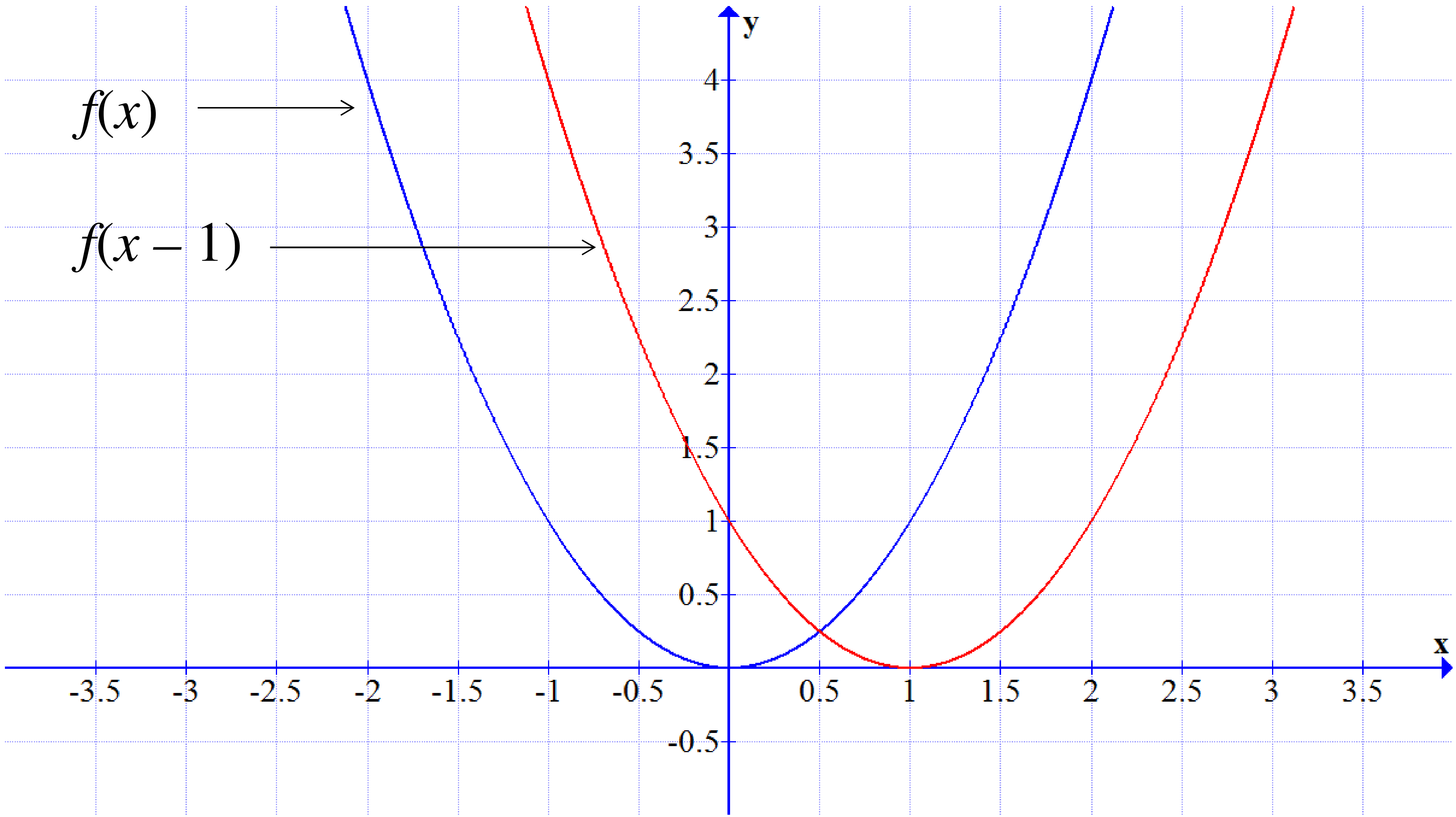
# Transformations: Graph 3 sequence



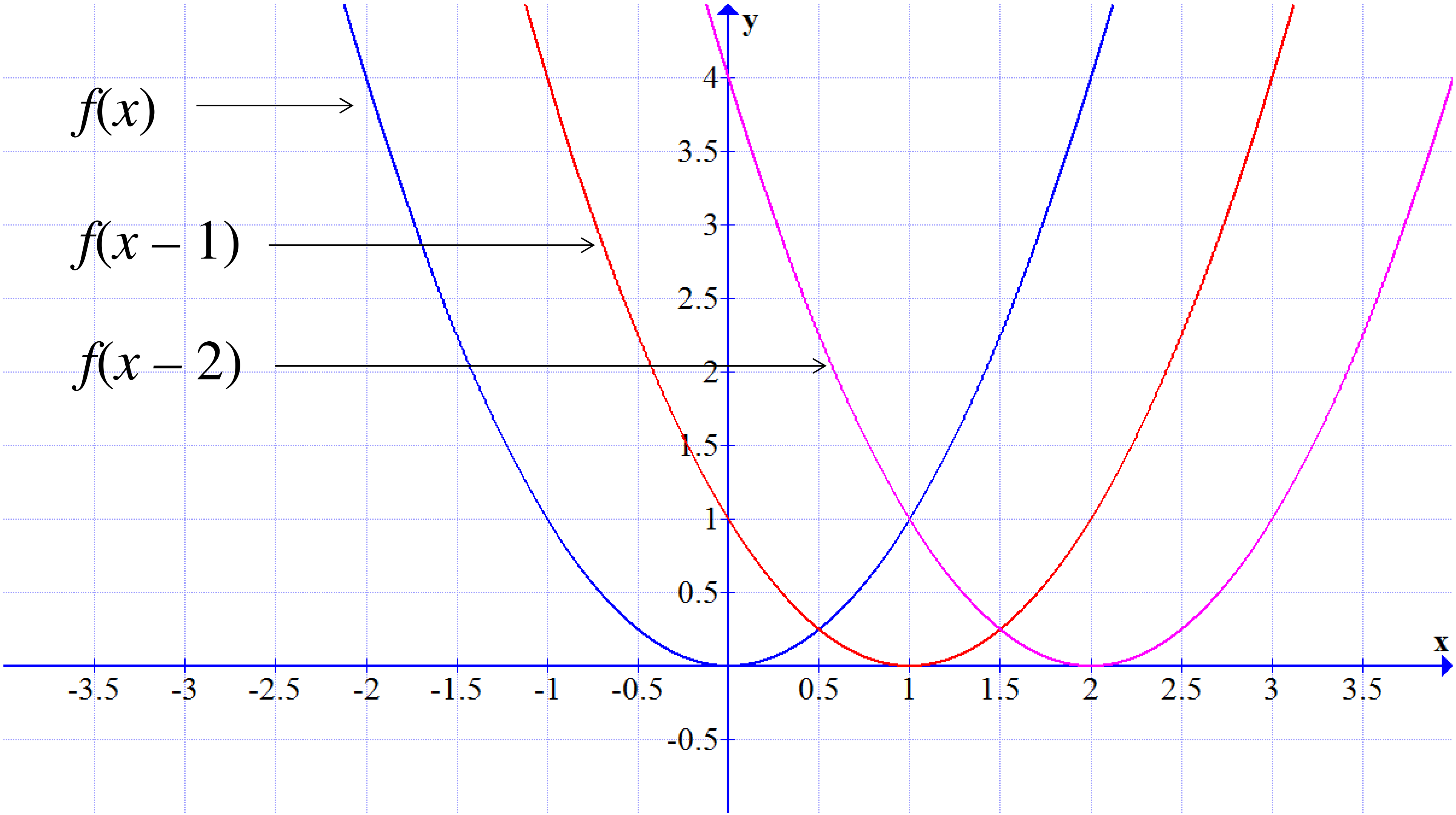
# Transformations: Graph 3 sequence



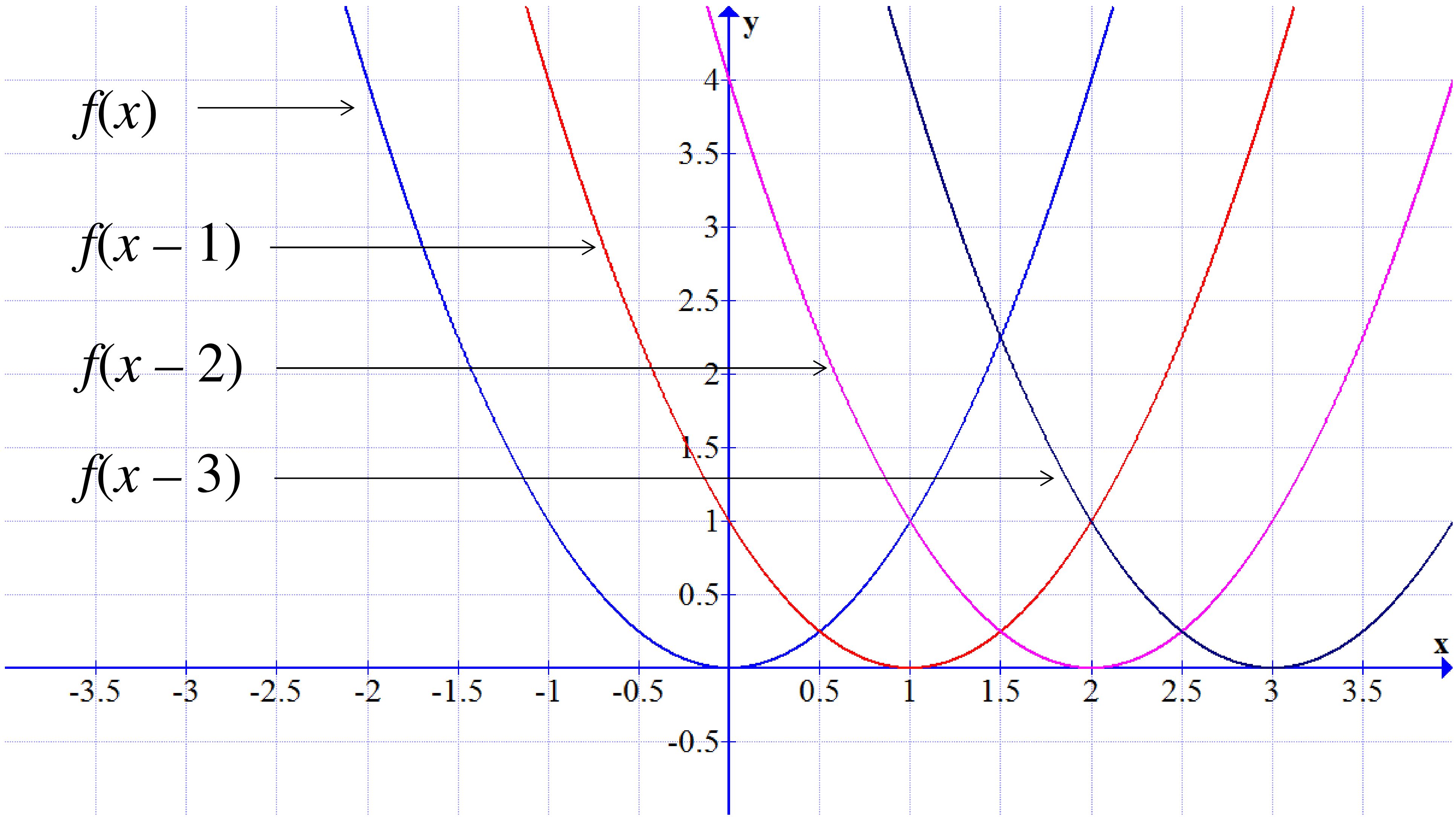
# Transformations: Graph 4 sequence



# Transformations: Graph 4 sequence



# Transformations: Graph 4 sequence



# Ways of speaking

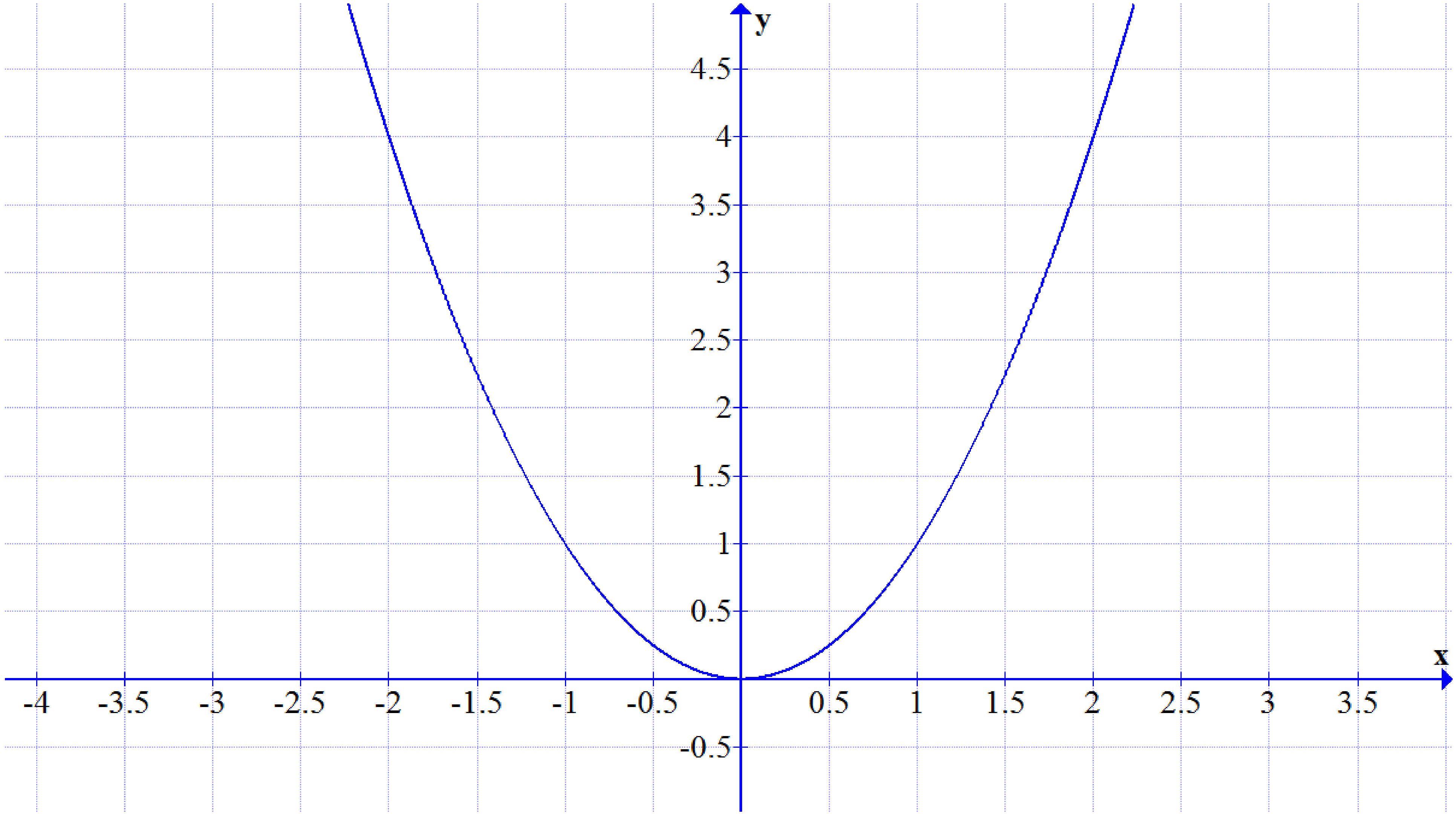
- *Question:* How are we going to describe the effects of the transformations shown in the previous examples?

- *Answer*

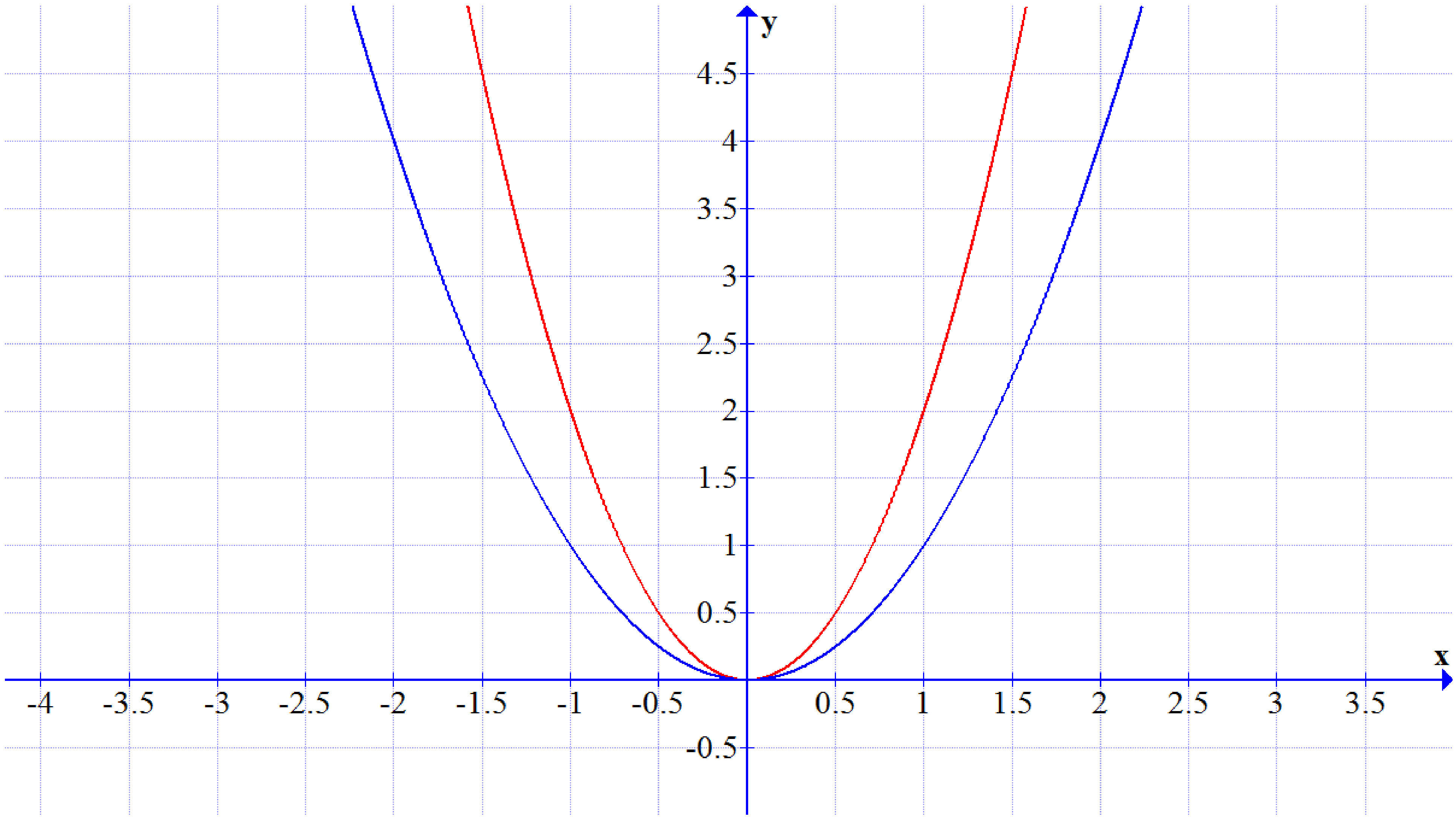
  See lesson

# Transformations: Graph 5 sequence

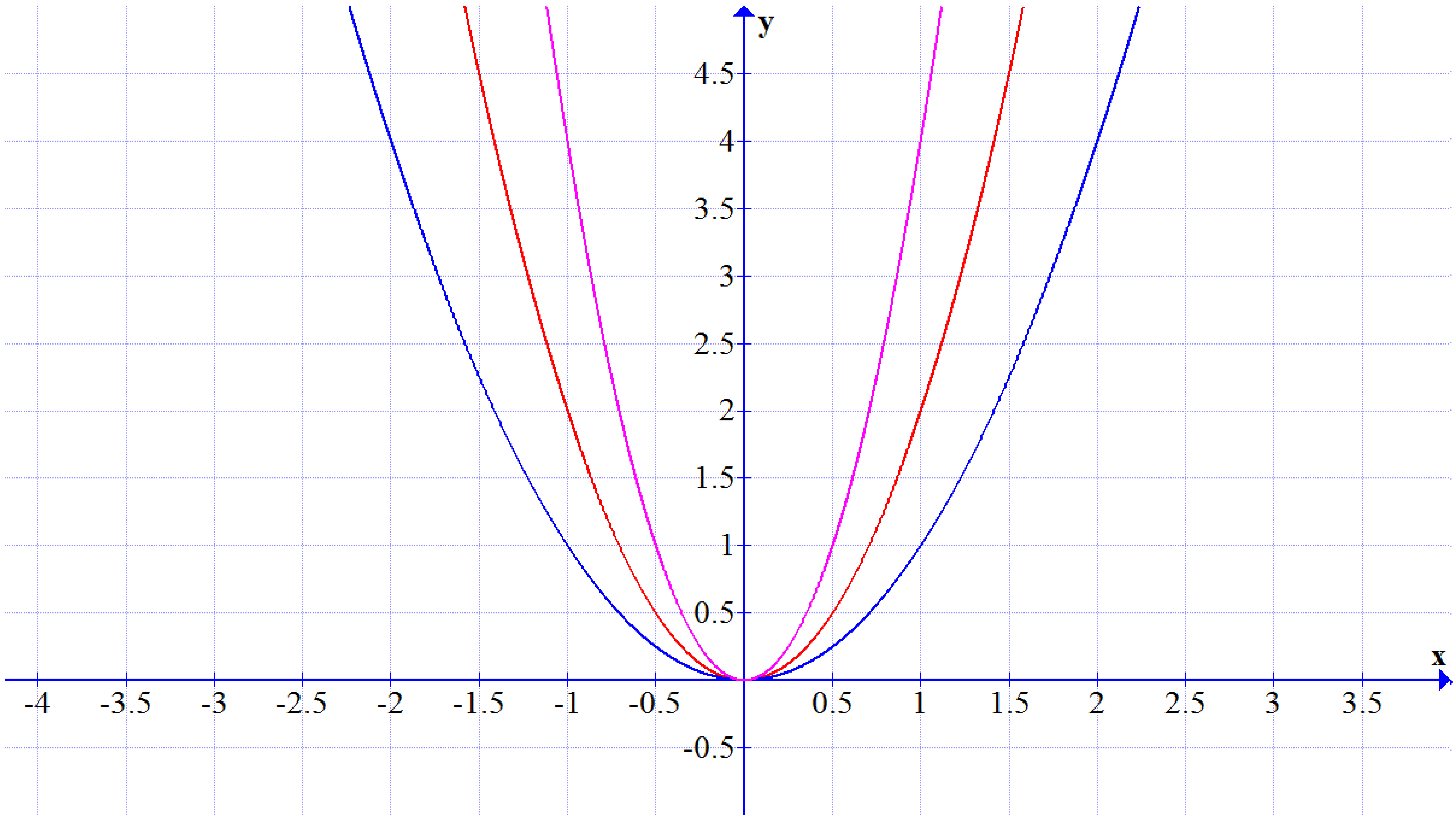
Let us look at the function $f(x) = x^2$, a graph of which is shown here



# Transformations: Graph 5 sequence

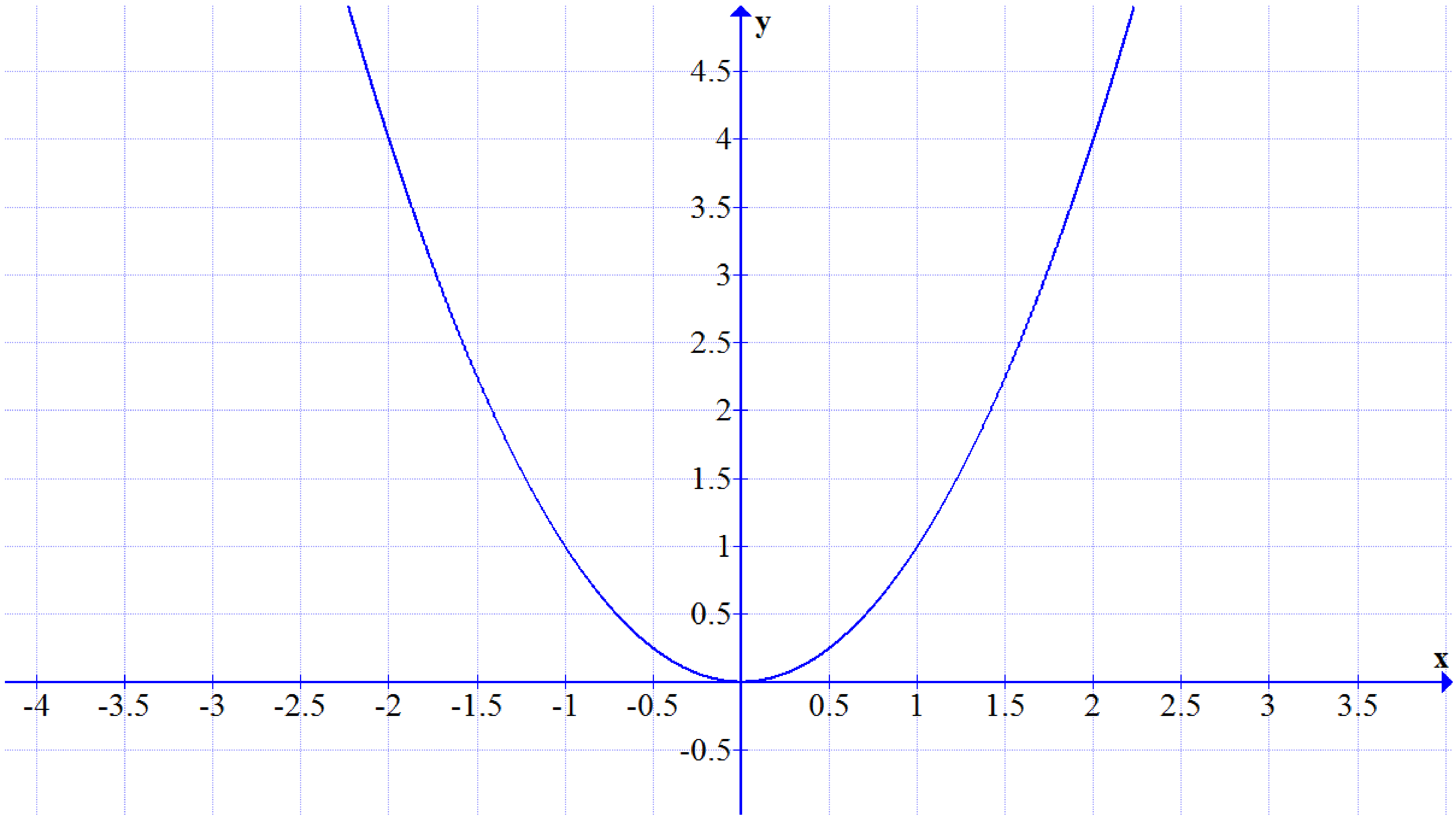


# Transformations: Graph 5 sequence

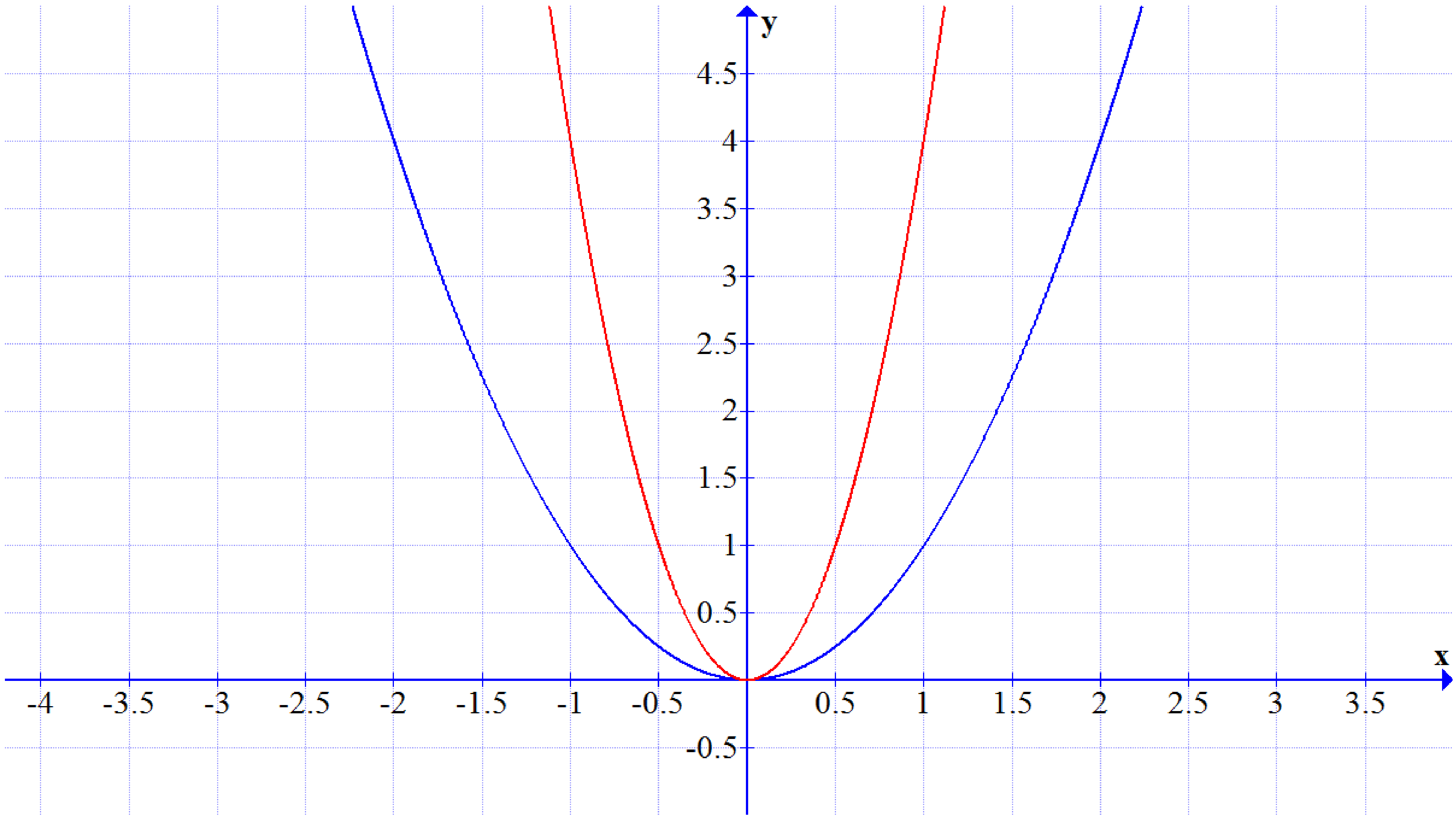


# Transformations: Graph 6 sequence

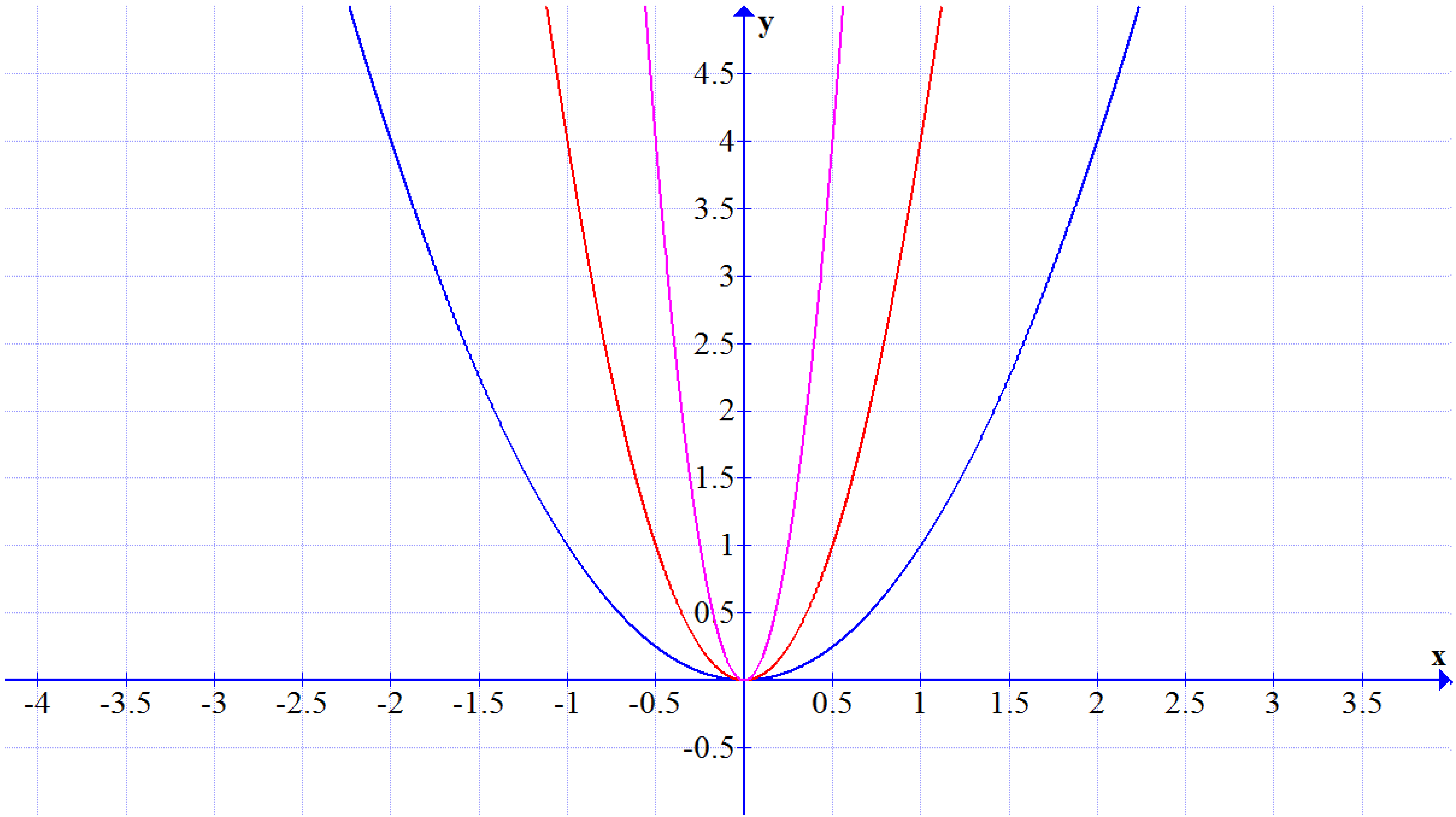
Let us look at the function $f(x) = x^2$, a graph of which is shown here



# Transformations: Graph 6 sequence



# Transformations: Graph 6 sequence



# Ways of speaking

- *Question:* How are we going to describe the effects of the transformations shown in the previous examples?

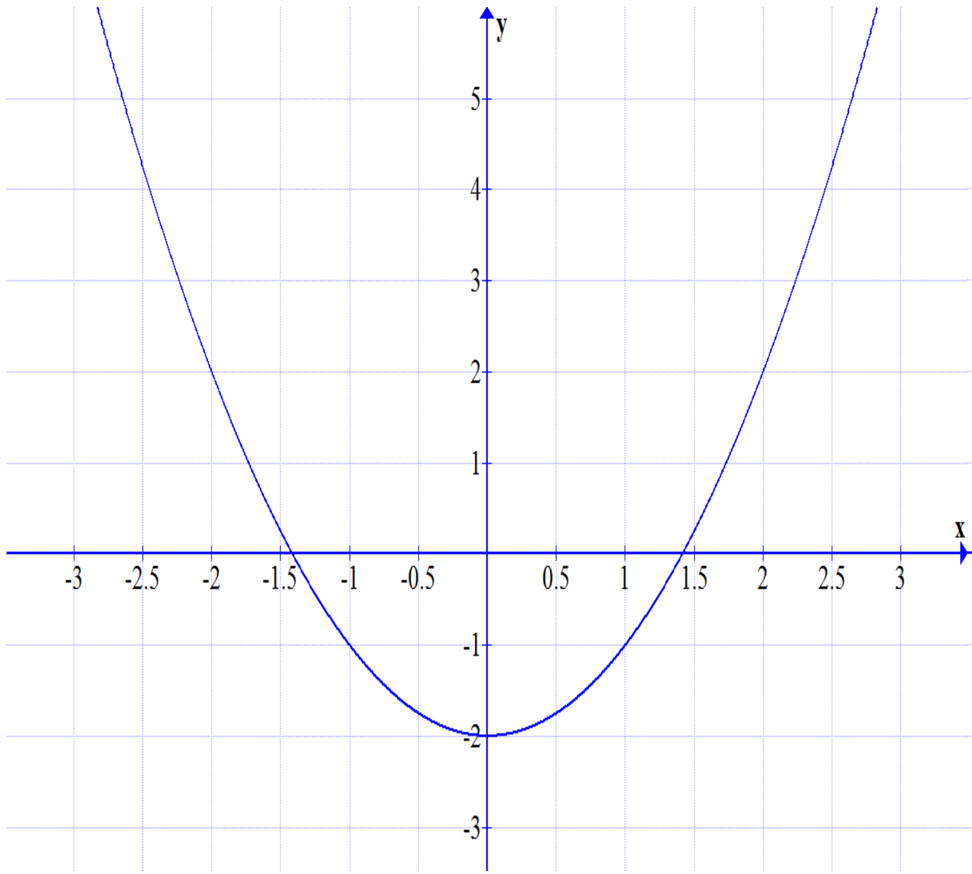
- *Answer*

  See lesson

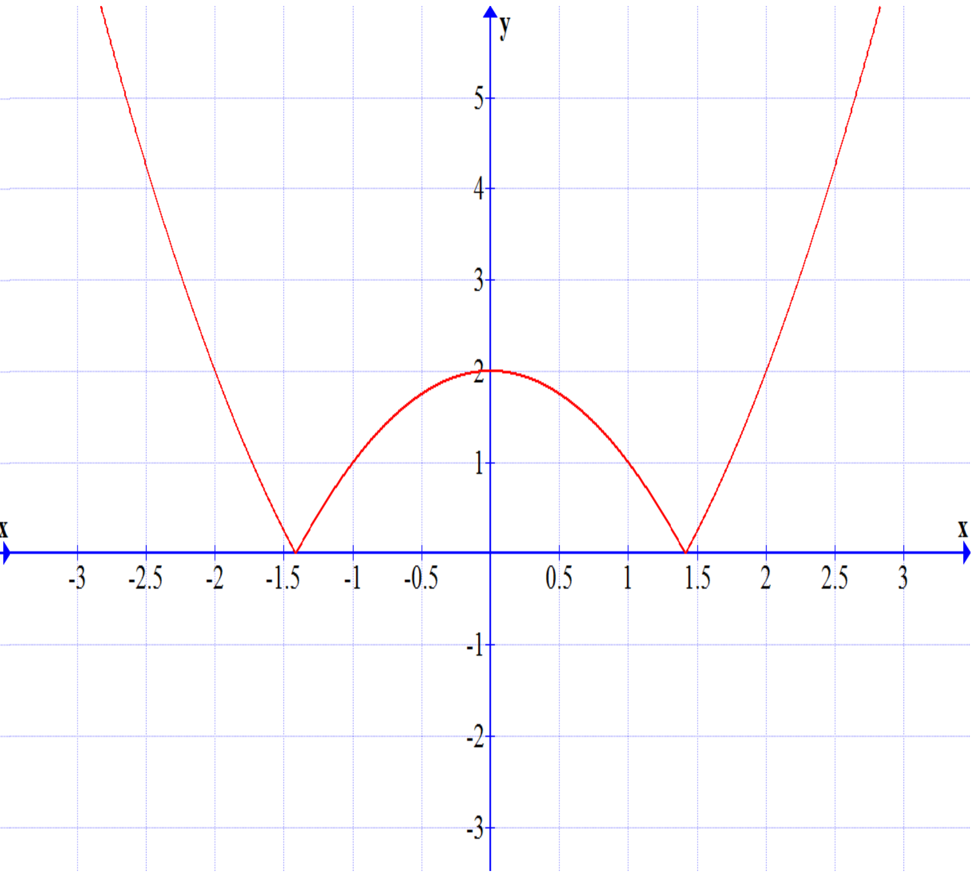
# Transformations: Graph 7 sequence

Let us look at the curve of the function $f(x) = |x^2 - 2|$

$f(x) = x^2 - 2$

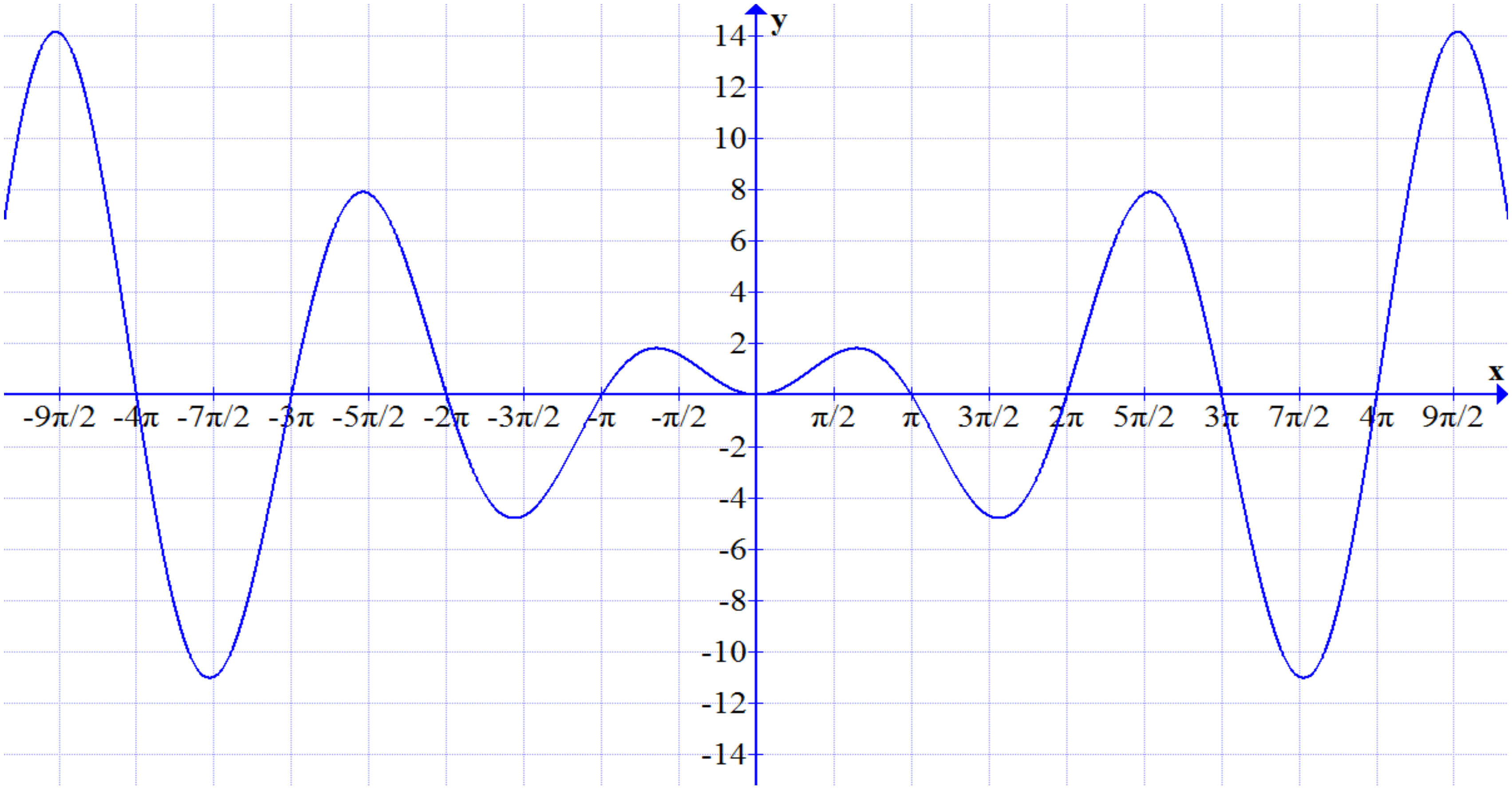



$|f(x)| = |x^2 - 2|$

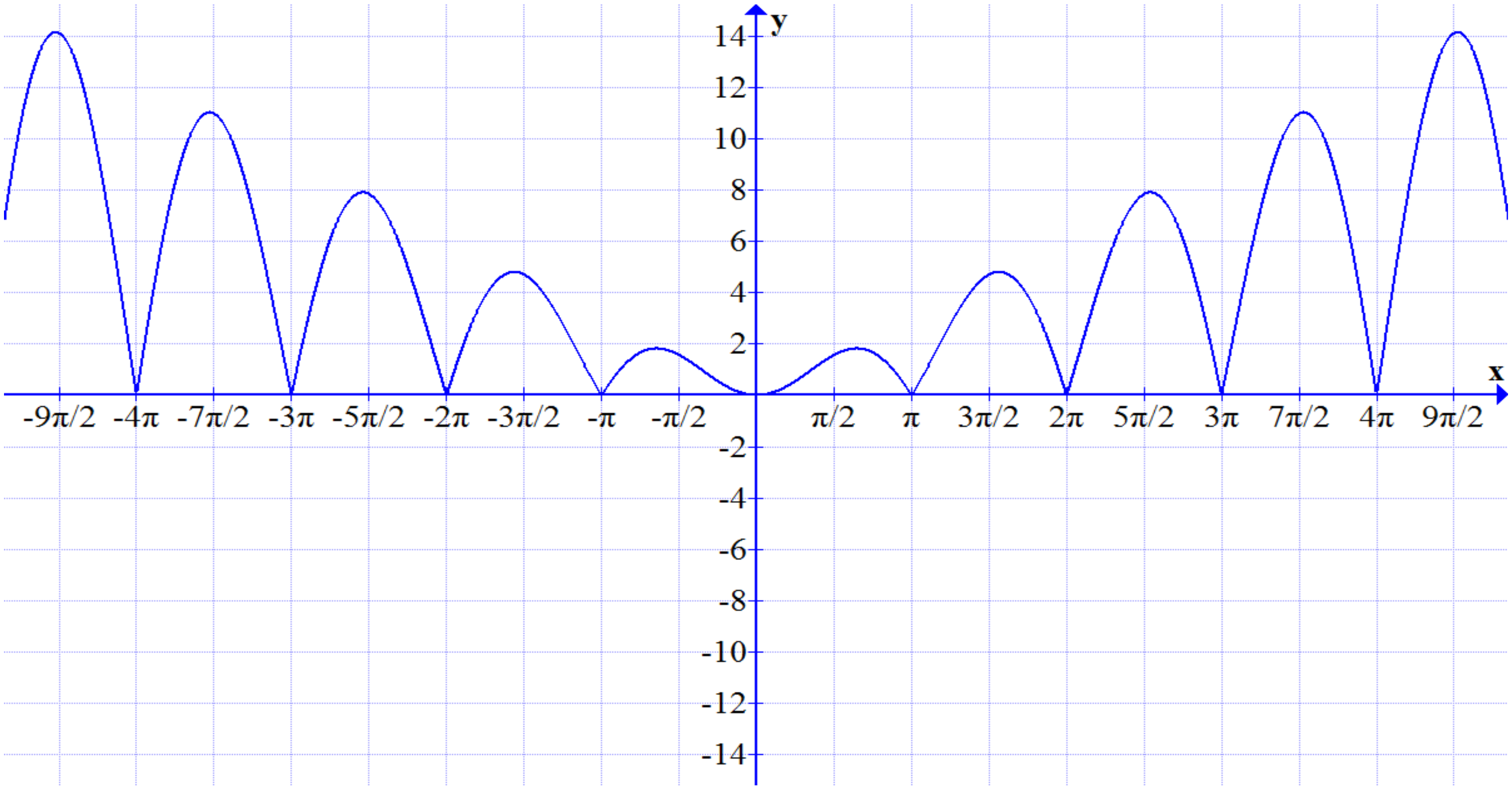


# Transformations: Graph 7 sequence

Another example: the curve of $f(x) = x\sin(x)$, is here:



# Transformations: Graph 7 sequence



# Ways of speaking

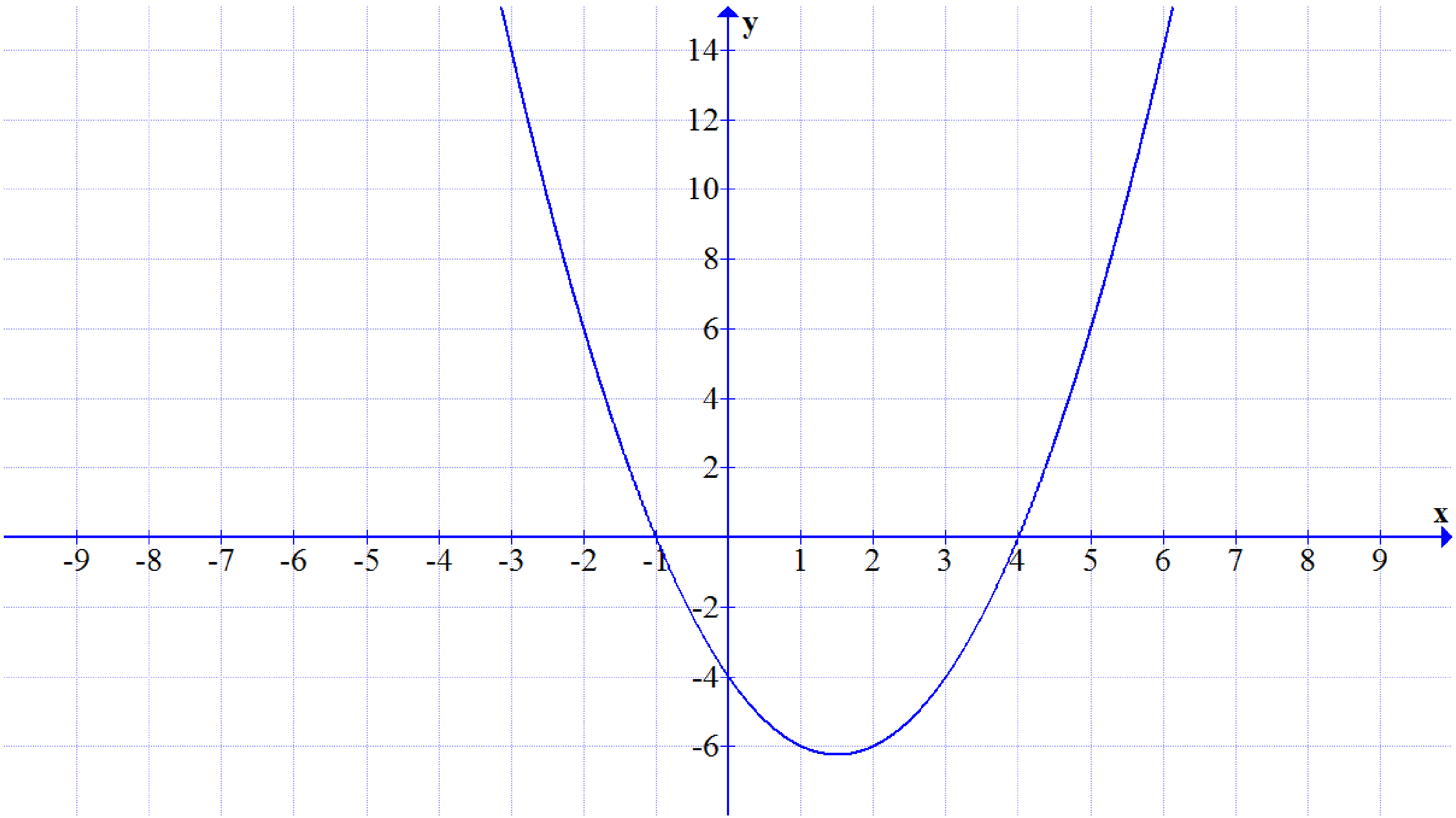
- *Question:* How are we going to describe the effects of the transformations shown in the previous examples?

- *Answer*

  See lesson

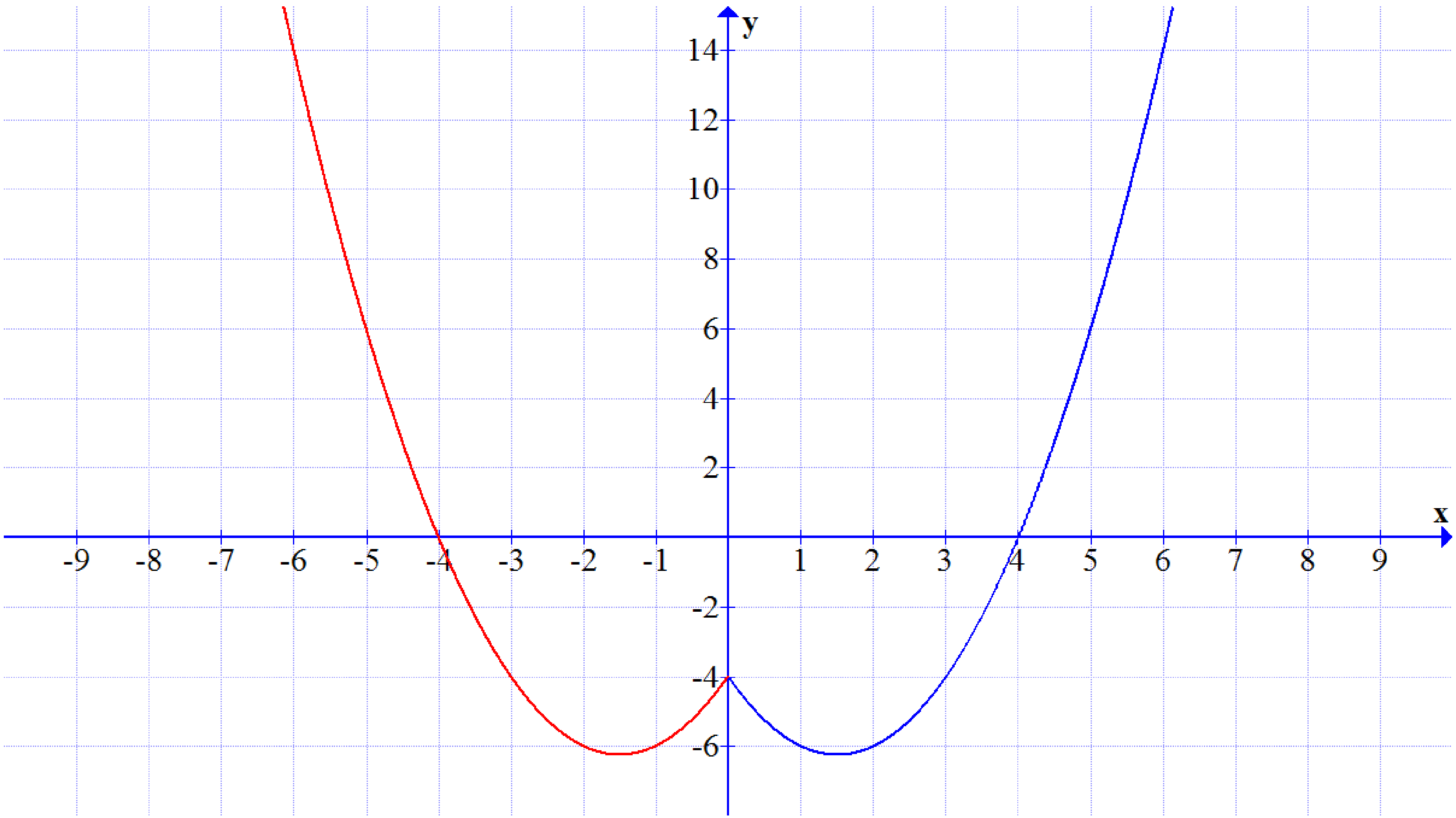
# Transformations: Graph 8 sequence

Let us look at the function $f(x) = x^2 - 3x - 4$, a graph of which is shown here



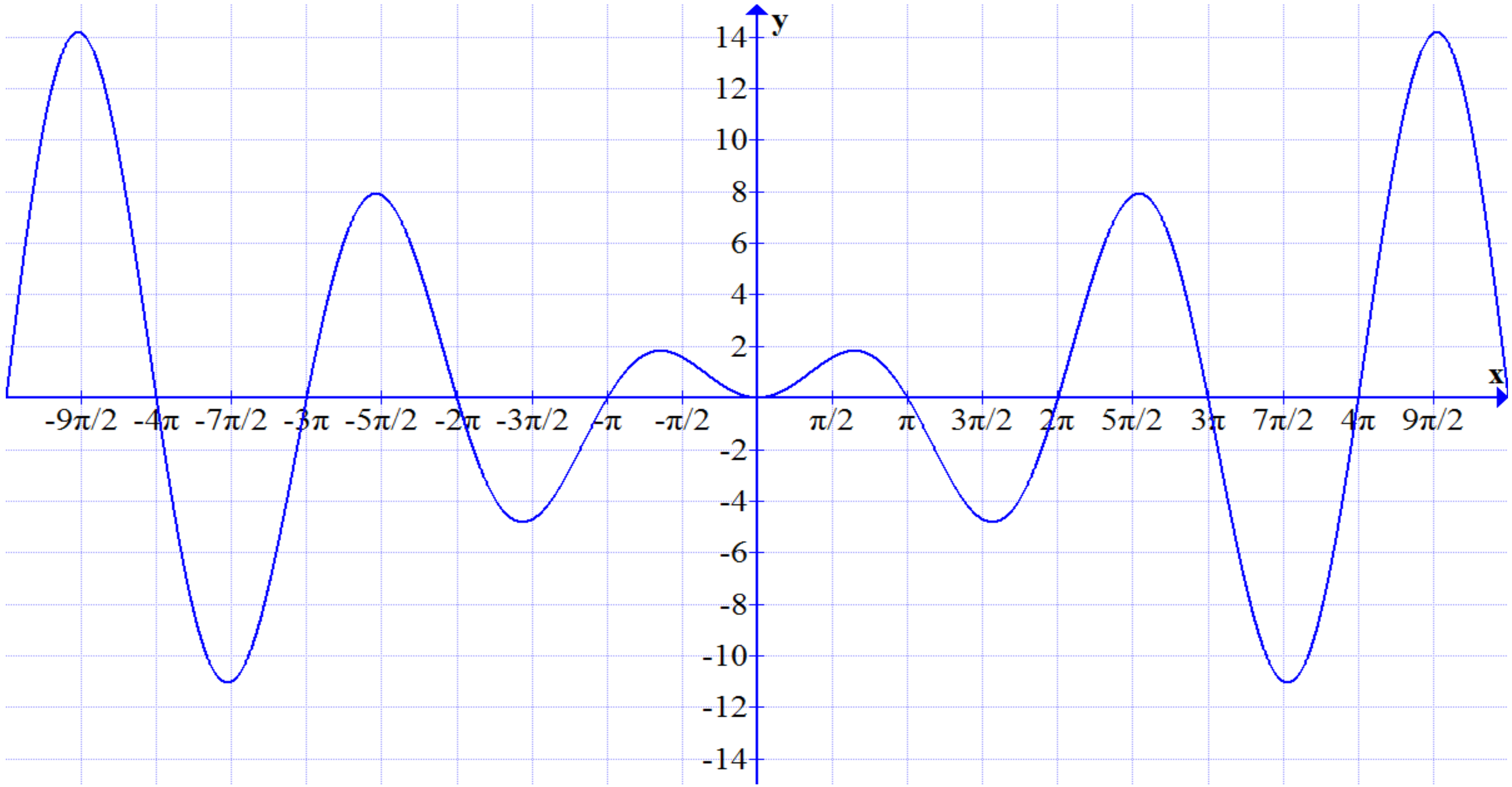
# Transformations: Graph 8 sequence

The effect of doing $f(|x|) = |x|^2 - 3|x| - 4$ is shown below:

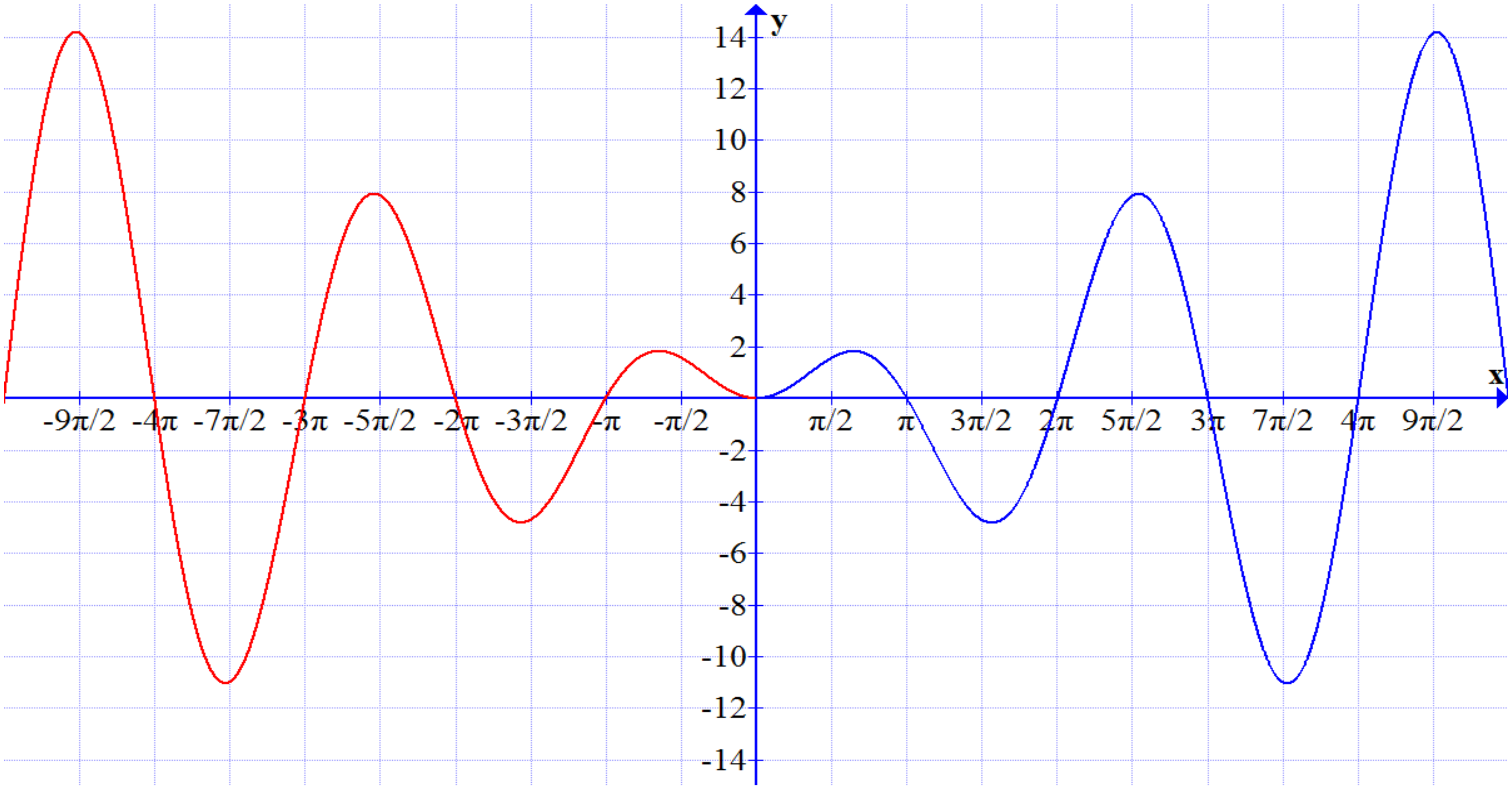


# Transformations: Graph 8 sequence

Another example: the curve of the function $f(x) = x.\sin(x)$, is shown here



# Transformations: Graph 8 sequence



# Transformations: Graph 8 sequence

- *Question:* How are we going to describe the effects of the transformations shown in the previous example?

- *Answer*

  See lesson

# Appendix

## 5) Transforming $f(x)$ to $|f(x)|$

So for $f(x)$ we will have $|-f(x)| = f(x)$.

This says that, whatever the current value of $f(x)$ we should make **all** $f(x)$ values positive.

What this means is that all values of $f(x)$ which are already positive will stay positive, and all values of $f(x)$ which are negative will *become* positive.

## 5) Transforming $f(x)$ to $|f(x)|$

Visually speaking this means that any part of the curve of $f(x)$ which lies underneath the $x$-axis (i.e. which is negative) will be reflected about the $x$-axis, to lie above the $x$-axis (i.e. to become positive)